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AN INTERNATIONAL REVIEW OF SPECTROSCOPY AND ASTRONOMICAL PHYSICS

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THE VARIATION WITH TIME OF THE CHARACTERISTICS OF A POTASSIUM PHOTO-ELECTRIC CELL AS TO SENSIBILITY ACCORDING TO WAVE-LENGTH

By HERBERT E. IVES

The measurements presented here were obtained during a study which was started with a twofold object: first, to try to work out a method of making photo-electric cells which would have the same characteristics of sensibility according to wave-length from cell to cell; secondly, to determine whether the spectral sensibility-curve remained constant with time and use.

A previous study,[1] part of a series planned to learn the possibilities of the photo-electric cell in photometry, established the fact that the distribution of sensibility through the spectrum varied greatly among cells made up in the manner usual at that time. If the proposition so to screen a cell as to make it equivalent to the eye is to have any practicability, it is essential that the cell's characteristics remain fixed; and if the manufacture of any considerable number is to be undertaken, it would be practically prohibitive to have to work out a separate correction for each cell. Hence the desirability of working out, if possible, a standardized cell of permanent characteristics.

[1] Ives, "Wave-Length Sensibility-Curves of Potassium Photo-Electric Cells," *Astrophysical Journal*, 40, 182, 1914.

The test for permanence giving disappointing results, as will be seen, the other portion of the study was necessarily abandoned,

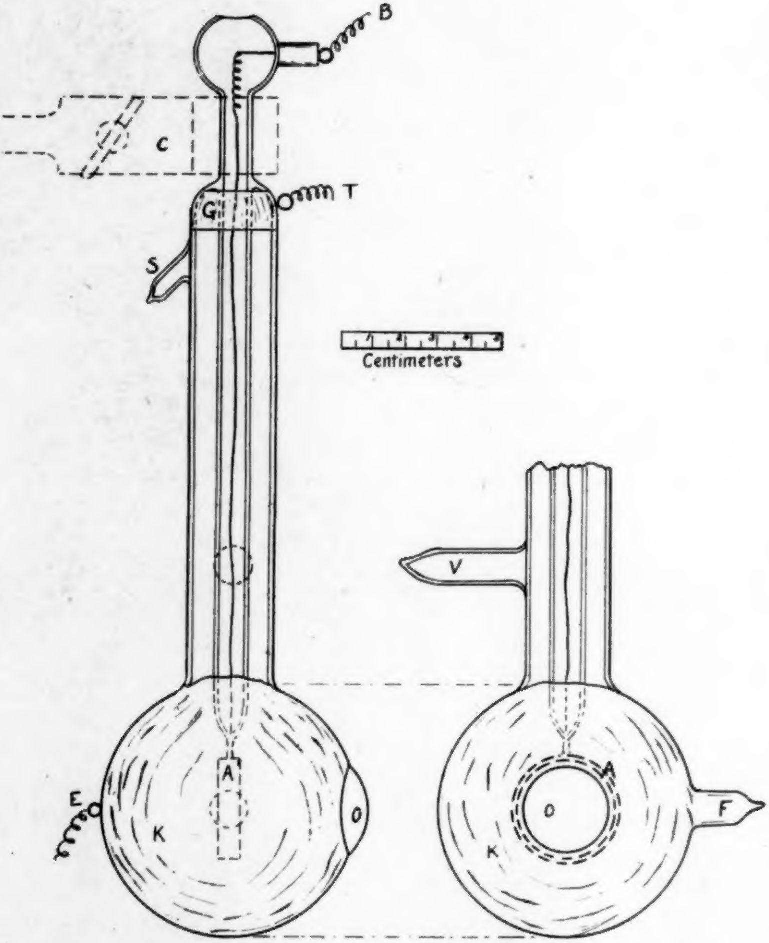

FIG. 1.—Photo-electric cell studied. *K*, Bulb whose inner surface coated with potassium forms the kathode; *A*, Anode; *E*, Electrometer connection; *O*, Opening for the admission of light; *F*, Filling tube; *V*, Evacuating tube; *S*, Tube used for silvering inner guard-ring; *G*, Guard-ring formed by interior and exterior silvering; *T*, Earth connection for guard-ring; *B*, Battery connection; *C*, Supporting clamp.

and work was limited to a single cell. Its design is shown in Fig. 1, with its accompanying legend. In general, it is made to conform

to the requirements for securing a rectilinear relationship between illumination and current as established by an earlier research,[1] namely, the reduction to a minimum of all surfaces on which charges can accumulate. The form adopted was the one described in the earlier paper as the Hughes design, that is, one in which the walls are covered with alkali metal, so that the whole cell approximates a black body. The choice of this form rather than of the Richardson design with the alkali metal at the center of the inclosure was dictated by the idea that the approximation to a black body might give the desired uniformity of characteristics from cell to cell. In order to secure sensibility all the way down to the red end of the spectrum, which is essential for photometric purposes, the cell was colored by a hydrogen glow discharge. This process was carried to the point where a maximum of sensibility was just passed, and all the conditions, such as voltage, pressure, etc., were carefully measured, so that they could be copied on subsequent cells. The resultant surface was dark blue in color. After the operation of coloring, the hydrogen was carefully pumped out, and helium was introduced to a pressure which gave the maximum sensibility as tested with the cell still on the pump. The curve of voltage-current of this cell was that characteristic of the gas-filled type—convex toward the voltage axis. The illumination-current relation was strictly rectilinear.

In testing the wave-length sensibility the light from a monochromatic illuminator was projected on to a piece of ground glass directly before the window of the cell, the idea being so to scatter the light that the average effect of all elements of the rather large area of potassium surface would be obtained.

The cell was mounted for test in a galvanized iron box which communicated with the electrometer inclosure, both being sealed air-tight with paraffin seals and well provided with drying material. During the whole period of the test neither cell, electrometer, nor monochromatic illuminator was disturbed in position. The source of light used was a standard 4.85 wpsc carbon lamp illuminating a ground glass before the slit of the monochromatic

[1] Ives, Dushman, and Karrer, "Factors Affecting the Relation between Photo-Electric Current and Illumination," *Astrophysical Journal*, **43**, 9, 1916.

illuminator. This lamp was used for no other purpose and provided a source of radiation of practically perfect uniformity of spectral distribution throughout the period of the test.

Measurements through the spectrum were made at first every day, then weekly, and then irregularly at longer intervals. After the first two weeks the cell was continuously illuminated, by an auxiliary lamp, with current running for a period of two weeks, to determine whether use accelerated the rate of change already detected. No effect directly chargeable to use was found. Five of the curves of wave-length sensibility for 2, 36, 63, 93, and 220 days (after which the arrangement of apparatus was disturbed) are shown in Fig. 2. These are just as obtained, not corrected either for the distribution of energy in the source or for the dispersion of the spectrometer. They are plotted to equal values at their maxima.

It is shown by these curves that the wave-length sensibility suffers a continuous change with time. The exact nature of the change cannot be inferred from these data, as it may be either an increase in the sensibility for the long waves or a decrease for the short waves. Thus, if the curves of Fig. 2 are plotted to equality in the yellow or yellow-green region of the spectrum, they all appear to be practically coincident until the blue-green is reached, when the variation shows as a progressive decrease of sensibility with time. This point could only be settled by preserving the constancy of the current-sensibility of the apparatus throughout the test and working with an applied voltage accurately the same on each trial, which was not attempted. For the main purpose of the test—establishing permanency or lack of it—it is of small importance whether the variation is due to an increase or a decrease of sensibility in any given region.

While the measurements give no information as to the cause of this variation, the suggestion may be hazarded that what is taking place is a gradual sintering of the highly sensitive and presumably unstable colloidal or hydride colored surface. This, as is well known, loses its characteristics if the hydrogen atmosphere, used to form it, is left in the cell. According to Elster and Geitel, an atmosphere of inert gas (helium, argon, neon) prevents this deterio-

ration. The present work indicates that some change still takes place with helium as the gas of the cell. From the standpoint of permanence alone, probably a very different result would be obtained if the coloring of the surface were dispensed with and

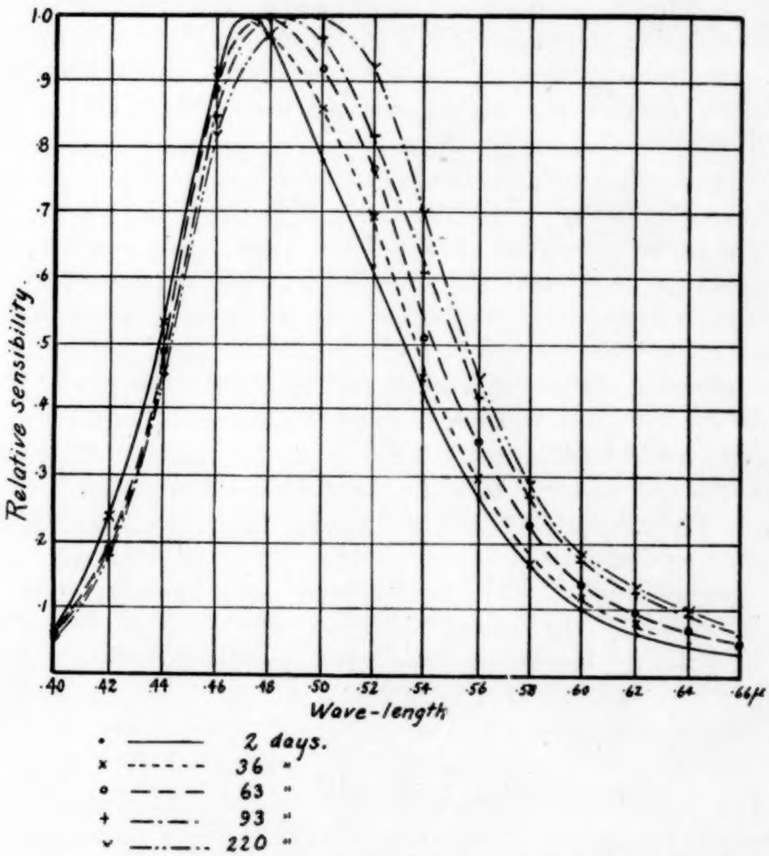

FIG. 2.—Variation with time of curve of sensibility according to wave-length

the potassium were simply made as pure and clean as possible by multiple distillation in vacuo. Pure potassium, however, is insensitive beyond 0.6 μ and so would be useless for the photometric purpose in view. Rubidium is sensitive farther into the red, but not far enough, while caesium, sensitive still farther, is too rare and of too low melting-point to be considered seriously.

It appears from this work that the prospect is not bright of making permanent cells of standardized characteristics. Consequently the practical fabrication of cells screened to imitate the human eye is not as yet feasible.

This investigation forms the conclusion of the series begun by the writer some years ago, aimed specifically at applying the photoelectric cell to photometry. For this purpose the cell should have certain characteristics. It should first of all exhibit a simple uniform relation between illumination and current. Next, it should have a curve of the relation between wave-length and sensibility which could be modified by screening so as to duplicate that of the average eye. The cell and its auxiliaries should, in addition, be sensitive, simple to use, and permanent. With regard to the first point, the writer's work[1] showed that in cells as ordinarily constructed, the relation of illumination and current was very erratic from cell to cell. Later[2] the explanation for this was found in the fact that whenever opportunity was offered for electrical charges to accumulate on free glass surfaces in the cell the current would be affected. Cells in which the relation of illumination and current could be altered at will were constructed. This led to the statement of the conditions necessary for a rectilinear relation of illumination and current, namely, the elimination, as far as possible, of such free glass surfaces, and to the production of cells possessing the desired relationship. The second point is covered by the measurements of curves of sensibility with wavelength of a series of cells, published some time ago,[3] and by the experimental work of the present paper.

The net result of this whole research is to show that the photoelectric cell is inapplicable at present to photometry, where by *photometry* is meant the measurement of luminous flux, that is, radiant flux evaluated according to its capacity to produce the

[1] Ives, "The Illumination-Current Relationship in Potassium Photo-Electric Cells," *Astrophysical Journal*, **39**, 428, 1914.

[2] Ives, Dushman, and Karrer, "Factors Affecting the Relation between Photo-Electric Current and Illumination," *Astrophysical Journal*, **43**, 9, 1916.

[3] Ives, "Wave-Length Sensibility-Curves of Potassium Photo-Electric Cells," *Astrophysical Journal*, **40**, 182, 1914.

sensation of light. The answer is quite different, however, in so far as ***radiometry*** is concerned, where under radiometry we include those measurements often treated under photometry, in which the characteristics of the radiometer as to sensibility according to wave-length do not enter. Such, for instance, are measurements of distribution of radiant flux, etc., for which the only requirement is the rectilinear relationship of illumination and current and adequate sensibility.

Among radiometric applications for which the properly constructed photo-electric cell is well suited may be noted, in particular, variations in intensity of certain types of stars, distribution curves of illuminants, densities of photographic plates, and finally spectro-radiometry in the visible and ultra-violet regions where the non-selective radiometers and the eye are too insensitive. For this latter purpose it is necessary, in order to get sufficient sensibility to make such measurements as those of diffuse reflecting power, to use an electrometer, or possibly a highly sensitive Thomson galvanometer. Some means of amplifying the current similar to those used in wireless telegraphy, to bring it into the range of the simple d'Arsonval galvanometer, would be a great boon. Some experiments made for the writer some time ago by Dr. A. W. Hull, of the General Electric Research Laboratory, showed the entire feasibility of amplifying photo-electric currents by the electron tube amplifiers developed in that laboratory. But while amplification of a thousand fold or more presented no difficulty, these amplifications were of currents already large enough to be measured by a sensitive d'Arsonval galvanometer. Small currents, of the order of magnitude measured by the electrometer, presented great difficulty, owing to leakage and other troubles in the amplifiers. Further work along this line is desirable.

Another improvement in the cell to make it applicable to spectro-radiometry would be to master the technique of producing a flat, extended curve of sensibility with wave-length, such as one of those accidentally found in one of the cells measured in the work quoted.[1] The very sharp maximum of the more usual type, such as that exhibited by the cell reported on in this paper, is

[1] *Ibid.*

undesirable, necessitating very narrow slit-widths to avoid corrections.

These points have been discussed in part to emphasize the necessary distinction between the very valuable characteristics of the photo-electric cell in *radiometry* and its failure in *photometry*. For the latter the only physical instrument so far developed is the screened non-selective radiometer, e.g., thermopile.[1]

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THE UNITED GAS IMPROVEMENT COMPANY
PHILADELPHIA, PA.
September 25, 1917

[1] Ives and Kingsbury, "Physical Photometry with a Thermopile Artificial Eye," *Physical Review*, **6**, 319, 1915.

THE PRINCIPLE OF GENERALIZED RELATIVITY AND THE DISPLACEMENT OF FRAUNHOFER LINES TOWARD THE RED[1]

By CHARLES E. ST. JOHN

According to Einstein's equivalence principle of generalized relativity, the lines in solar and stellar spectra should be displaced to the red when referred to the corresponding terrestrial lines. For any given frequency the magnitude of the effect depends upon the difference in gravitational potential between the gravitational field in which the emitting center is located and the terrestrial field where the radiation is received and measured. Einstein[2] deduces an approximate relation between the frequencies in the solar and terrestrial fields, namely,

$$(n_0-n)/n_0=2\times 10^{-6},$$

or, postulating the constant space-velocity of light,

$$\lambda-\lambda_0=2\times 10^{-6}\lambda.$$

The calculated displacement of two parts in a million is well within the possibilities of observation, being 0.010 A for λ 5000. A less approximate and perhaps more objective statement is, "at the surface of the sun the displacement is equivalent to the Doppler displacement produced by a radial velocity of 0.634 km per sec."[3]

As the occurrence or non-occurence of such displacements is of fundamental importance in the theory of relativity and in the interpretation of observations of solar and stellar spectra, definite results from an investigation carried on with powerful instruments at command would have a double significance. The present contribution presents the details of such an investigation in which the primary object has been to determine what consideration must

[1] *Contributions from the Mount Wilson Solar Observatory*, No. 138.

[2] *Annalen der Physik*, **35**, 905, 1911.

[3] A. S. Eddington, *Monthly Notices*, **77**, 380, 1917.

be given to this deduction from the equivalence principle of generalized relativity in the discussion of observations of solar spectra.

INSTRUMENTS AND METHODS

The 30-foot Littrow spectrograph and the 60-foot tower telescope on Mount Wilson were used in obtaining the main portion of the data. A grating of high resolving power and excellent definition was at our disposal through the kindness of the Physical Laboratory of Johns Hopkins University, the ruled surface being 114×160 mm and the number of lines 95,000. The third and fourth orders with scales of 0.56 A and 0.36 A per mm were used.

A common defect in narrow comparison spectra obtained by the usual occulting arrangement at the slit is the unsymmetrical ends of the arc lines and the irregular edges of the solar spectra. This disturbing element was eliminated, and the ease and accuracy of the measurements were increased by stretching a violin string on each side of the central strip of spectrum and close to the photographic film. This also removed the effect of a slight astigmatism without involving the movement of an occulting shutter. Other possible instrumental displacements were avoided by making all comparison exposures simultaneous, the relative intensities being controlled by a rotating sector.[1] The data in particularly important cases are the means of the closely agreeing measures made by three observers, and in all instances depend upon at least two.

SELECTION OF OBSERVATIONAL MATERIAL

Besides the principle of relativity, three other causes have been suggested which might account for a displacement of the Fraunhofer lines in the direction of longer wave-lengths—namely, differences of prèssure, motion in the line of sight, and anomalous refraction. While anomalous refraction may produce sporadic effects under occasionally favorable density-gradients in the solar atmosphere, the conclusion from investigations and observations at this observatory is that, within the present limit of precision of measurement, the positions of the Fraunhofer lines in the spectrum

[1] St. John and Babcock, *Mt. Wilson Contr.*, No. 106; *Astrophysical Journal*, **42**, 231, 1915.

of the solar disk are not systematically affected by anomalous refraction.[1] Any effect due to the difference in gravitational potential between the solar and terrestrial fields may be freed from that due to pressure by using lines whose pressure-shifts are negligible, such as those in the bands of the carbon arc at λ 3883 variously assigned to carbon, cyanogen, and nitrogen;[2] it may be freed from the influence of motion in the line of sight by observations upon the same lines at the sun's polar limb.

The richness of the ultra-violet Fraunhofer spectrum and the multiplicity of lines in the carbon-arc spectrum make the selection of suitable lines a matter of grave consideration from the point of view both of precision in measurement and of contamination by blends with lines sensitive to pressure changes. From an examination of high-dispersion spectrograms of excellent definition lines were selected whose separation from neighboring lines was sufficient to assure that the measurements would be free from systematic errors. A previous investigation had revealed such errors and shown that their magnitude depends upon the degree of proximity and the character and intensity of the adjacent lines; it also furnished data for determining for spectrographs of the dispersion and resolving power used in this investigation the limiting separation from neighboring lines within which measurements become unreliable.[3]

The lines finally selected were those of the nitrogen (cyanogen) bands listed in Tables I and II. Since they show wide variations in character and surroundings, an estimate was made of the weight to be assigned to results for individual lines. This estimate, based upon the appearance of the lines in both solar and arc spectra, is the observer's a priori judgment whether the measurements would have high, medium, or low weight. These estimated weights are entered in the appropriate tables.

[1] St. John, *Mt. Wilson Contr.*, Nos. 93 and 123; *Astrophysical Journal*, **41**, 28, 1915, and **44**, 311, 1916.

[2] The observations of Grotrian and Runge show that these bands are produced under experimental conditions that apparently preclude the presence of carbon, and that the essential condition is the presence of nitrogen in the absence of oxygen (*Physikalische Zeitschrift*, **15**, 545, 1914).

[3] St. John and Ware, *Mt. Wilson Contr.*, No. 120; *Astrophysical Journal*, **44**, 15, 1916.

THE OBSERVATIONS

A. The arc wave-lengths.—The wave-lengths of the lines under investigation were determined in the arc in terms of the same international iron standards and under the same instrumental conditions as were later used for their solar wave-lengths. The agreement with the wave-lengths of Uhler and Patterson was so close that the mean was taken as the probable value. For a few difficult cases, in which evidently neighboring lines might have affected the measures, appeal was made to the difference, Rowland *minus* International, to decide which was the more probable wave-length of the arc-line. Uhler and Patterson[1] consider their wave-lengths for sharp lines on a clear background to be correct in absolute value to ±0.005 A. This appears to be an underestimate of the accuracy, as the mean difference, Uhler and Patterson *minus* Mount Wilson, taken without regard to sign, is only 0.003 A even when difficult lines are included in the comparison. The decimals of the adopted values of the wave-lengths are in the fourth column of Table I.

B. Sun—arc displacements.—The critical questions are the behavior of the investigated lines at the center of the solar disk and the relation between center and limb displacements. Owing to the importance of the possible results and the limited observational material available, the questions have been approached from several directions:

1. At the center: (*a*) The center—arc displacements were directly determined from simultaneous exposures on the sun and the equatorial sections of a carbon arc 6–7 mm long, fed by a current of 6–7 amperes. These and all other comparisons between solar and terrestrial sources have been corrected for the earth's motion. The results are in the first column of Table I.

(*b*) The absolute wave-lengths of the solar lines were obtained from simultaneous exposures on the sun and on the iron arc, in terms of the same iron standards as were used for the arc wave-lengths. The differences between the solar and arc wave-lengths in the third and fourth columns of Table I furnish an independent determination of the center-arc displacements given in the fifth column.

[1] *Astrophysical Journal*, **42**, 438, 1915.

(*c*) The stable iron lines are displaced to the red in the solar spectrum. The positive differences, Rowland wave-length *minus* International wave-length, for such lines when decreased by the corresponding displacements center *minus* arc give the difference R−I that would be shown by lines whose solar and arc wave-lengths are equal. For the spectral region under consideration the mean value of this difference is 0.137A, which will be referred to as the "standard" R−I. Lines for which R−I is greater than 0.137A are displaced to the red in the sun; those for which it is less are displaced to the violet, the excess and deficit being the measures of the corresponding center−arc displacements. Hence a

TABLE I

RELATIVE DISPLACEMENTS OF SOLAR AND ARC LINES OF THE NITROGEN (CYANOGEN) BANDS AT CENTER OF SUN

SECTION A. LINES OF SOLAR INTENSITY 00–1

(*a*) Sun−Arc	λ Center of Sun		λ Arc I.U.	(*b*) λ Sun *minus* λ Arc	(*c*) Rowland *minus* λ Arc	Intensity Rowland	Series	Weight
	R-System	I.U.						
+0.002..	3819.200	.061	.062	−0.001	0.138	1 Nd	B_1	High
+ .002..	3825.260	.126	.125	+ .001	.135	00	C_1	High
− .002..	3833.744	.610	.610	.000	.134	0	C_1	Medium
.000..	3842.780	.644	.648	− .004	.132	0	......	High
+ .001..	3845.152	.021	.019	+ .002	.133	1	B_1	High
.000..	3846.667	.534	.536	− .002	.131	00	......	High
.000..	3848.981	.848	.848	.000	.133	1 N	B_1	High
+ .002..	3850.784	.652	.648	+ .004	.136	0	B_1	Low
.000..	3854.192	.062	.063	− .001	.129	0	B_1	High
+ .002..	3854.988	.855	.854	+ .001	.134	1	B_1	Low
.000..	3857.058	.926	.922	+ .004	.136	0	......	High
+ .002..	3857.288	.156	.158	− .002	.130	1	B_1	High
+ .002..	3858.723	.591	.592	− .001	.131	0	B_1	Medium
− .002..	3867.116	.980	.983	− .003	.133	0	A_2	High
+ .002..	3867.200	.064	.061	+ .003	.139	0	A_2	High
− .003..	3867.908	.776	.779	− .003	.129	1	A_2	High
.000..	3868.542	.410	.410	.000	.132	1	A_2	Low
+ .004..	3868.624	.489	.486	+ .003	.138	00	B_2	Low
.000..	3868.702	.570	.572	− .002	.130	0	A_2	High
+ .003..	3868.782	.648	.644	+ .004	.138	00	A_2	Low
− .002..	3872.312	.178	.181	− .003	.131	0	A_2	High
− .001..	3876.448	.312	.315	− .003	.133	0	A_1	High
− .001..	3877.482	.346	.351	− .005	.131	1	A_1	High
+ .001..	3879.716	.580	.579	+ .001	.137	1	A_1	Medium
+0.003..	3880.102	.966	.964	+0.002	0.138	1	A_1	Low
+0.0006.	....	....	....	−0.0002	0.134	....	....	....

Average number of measures per line, 13. Average mean deviation per line, 0.002 A.

TABLE I—*Continued*

SECTION B. LINES OF SOLAR INTENSITY 2–4 AND PARTIALLY RESOLVED PAIRS

(a) Sun−Arc	λ Center of Sun R-System	λ Center of Sun I.U.	λ Arc I.U.	(b) λ Sun *minus* λ Arc	(c) Rowland *minus* λ Arc	Intensity Rowland	Series	Weight
	R.346					1	A_1	
+0.001..	3819.384	.241	.244	−0.003	0.140			Low
	R.412					1	A_1	
	R.406					0	A_1	
+ .001..	3822.435	.290	.290	.000	.145			Low
	R.470					0	A_1	
	R.745					0	B_1	
.000..	3830.774	.630	.637	− .007	.137			Low
	R.801					0	B_1	
+ .004..	3831.174	.038	.034	+ .004	.140	3	A_1d	Low
	R.639					1	A_1	
+ .005..	3836.660	.520	.516	+ .004	.144			Low
	R.689					1	A_1	
+ .005..	3844.377	.238	.235	+ .003	.142	4	A_1d	Low
− .002..	3846.131	.993	.999	− .006	.132	2	B_1	High
	R.777					1	A_1	
.000..	3846.796	.657	.656	+ .001	.140			Low
	R.814					1	A_1	
+ .004..	3851.426	.292	.286	+ .006	.140	2 N	A_1d	Medium
+ .006..	3852.541	.408	.402	+ .006	.139	2 N	A_1d	Low
− .003..	3853.621	.487	.491	− .004	.130	2 N	A_1d	Low
+ .006..	3856.800	.667	.665	+ .002	.135	2 N	A_1d	Low
+ .004..	3858.822	.692	.684	+ .008	.138	2 N	A_1d	Medium
+ .002..	3862.626	.493	.489	+ .004	.137	2	A_1b	Low
+ .004..	3863.534	.402	.399	+ .003	.135	3 N	A_1b	Low
+ .004..	3864.438	.308	.303	+ .005	.135	3	A_1	High
+ .003..	3865.284	.150	.150	.000	.134	3	A_1sB_1	Low
0.000..	3866.125	.989	.993	−0.004	0.132	3 N	A_1sB_2	Low
+0.0024	....	....	....	+0.0012	0.138	....	....	....

Average number of measures per line, 22. Average mean deviation per line, 0.003 A.

comparison of this "standard" value with R−I for the carbon-arc or nitrogen bands furnishes a method independent of the direct center−arc determinations.

A preliminary examination indicated that as a class the carbon-arc lines were not appreciably displaced to the red in the solar spectrum. A possible explanation of this systematic difference in the behavior of the iron and the band lines was that the Rowland wave-lengths for the two classes of lines did not form a homogeneous system. It is known that Rowland made an intensive investigation of these band lines, but it is not clear how their wave-lengths were connected with his standards. The solar wave-lengths

of the lines under investigation have therefore been redetermined in the Rowland system, the Rowland values for the neighboring iron lines being used as standards. There are no systematic differences between the Rowland and Mount Wilson measurements, the sums of the positive and negative differences for 57 lines being +0.086 and −0.094 A. The means of the two measurements have been taken as the wave-lengths in the Rowland system and are entered in the second column of Table I. In the sixth column are given the differences, Rowland *minus* International. For the lines of solar intensity 00–1 a mean displacement of 0.003 A to the violet is indicated, and for those of intensity 2–4 a displacement of 0.001 A to the red.

(*d*) A fourth independent method of determining the center−arc shifts is based upon the observations detailed below, which refer to the limb and its relation to the center, as the displacements, limb−arc, decreased by the corresponding limb−center values, are another measure of the center−arc shifts.

2. At the sun's limb: (*a*) The absolute wave-lengths of the investigated solar lines were measured at the limb in terms of the iron standards used for the arc wave-lengths. The results are in the second column of Table II. Simultaneous exposures were made on the center of the iron arc and the sun's limb in latitude 90° at a point distant from the edge one-eightieth of the diameter of the solar image. The displacements at the limb are then λ limb−λ arc as given in the third column.

(*b*) A check is furnished by increasing the center−arc displacements by the limb−center shifts independently obtained. The latter were found by the method used by Adams,[1] and also from simultaneous exposures on the center and on a point on the limb in latitude 90°. The means of the closely agreeing results given by these two methods are entered in the fifth column of Table II under the heading "Limb *minus* Center."

It is to be remarked that the Rowland intensities given in Tables I and II should in many cases be modified. It is especially difficult to estimate the intensities of the components of close pairs. In general, the values assigned are too low. For example, the

[1] *Mt. Wilson Contr.*, No. 43; *Astrophysical Journal*, **31**, 30, 1910.

double lines $\begin{Bmatrix}3836.639, 1\\3836.689, 1\end{Bmatrix}$ and $\begin{Bmatrix}3846.777, 1\\3846.814, 1\end{Bmatrix}$ are like 3844.378, *4d*, in appearance and intensity in both arc and solar spectra. Though quite differently estimated by Rowland, they are placed in the same section of these tables, and, similarly, all partially resolved doublets measured as single lines are classed with nominally stronger lines, which in reality are more or less close pairs of similar lines.

The lines under investigation are within a region of 60 A, in which Rowland catalogues about 250 lines belonging to the bands. In equal regions to the red and violet of the bands there are 400 and 450 metallic lines, respectively. If the region λ 3819–λ 3880 is equally rich in such lines, the probability of blends among the 250 band lines is high. It is highest for broad lines formed by the coalescing of the components of doublets, higher even than for narrower lines of greater inherent intensity; moreover, the errors of measurement may be systematic, as frequently the compound lines are unsymmetrical. As the probable occurrence of blends is least for the

TABLE II

BEHAVIOR OF THE LINES OF THE NITROGEN (CYANOGEN) BANDS AT THE SUN'S LIMB

SECTION A. LINES OF INTENSITY 00–1

Lines	(a) λ Limb	λ Limb *minus* λ Arc	λ Limb *minus* λ Center	λ Limb *minus* λ Center	Intensity	Series	Weight
3819.197 ...	.064	+0.002	+0.003	+0.003	1 N	B_1	High
3825.256 ...	.126	+ .001	.000	+ .003	00	C_1	Low
3833.744 ...	.611	+ .001	+ .001	+ .004	0	C_1	Medium
3842.779 ...	.646	− .002	+ .002	+ .005	0	......	High
3845.149 ...	.023	+ .004	+ .002	+ .002	1	B_1	Medium
3846.666 ...	.536	.000	+ .002	.000	00	......	Medium
3848.979 ...	.846	− .002	− .002	− .002	1 N	B_1	High
3854.191 ...	.065	+ .002	+ .003	+ .004	0	B_1	High
3854.989 ...	.855	+ .001	.000	+ .001	1	B_1	Low
3857.063 ...	.927	+ .005	+ .001	+ .002	0	......	Medium
3857.288 ...	.156	− .002	.000	+ .008	1	B_1	Medium
3867.118 ...	.982	− .001	+ .002	+ .001	0	A_2	Low
3867.906 ...	.779	.000	+ .003	+ .002	1	A_2	High
3876.448 ...	.310	− .005	− .002	.000	0	A_1	High
3877.481 ...	.346	− .005	.000	− .002	1	A_1	High
3879.716 ...	.579	.000	− .001	− .002	1	A_1	Medium
3880.105 ...	.964	0.000	−0.002	−0.004	1	A_1	Medium
....	....	0.0000	+0.0007	+0.0015	....	......	....

TABLE II—*Continued*

SECTION B. LINES OF INTENSITY 2–4 AND PARTIALLY RESOLVED PAIRS

Lines	(a) λ Limb	λ Limb *minus* λ Arc	λ Limb *minus* λ Center	λ Limb *minus* λ Center	Intensity	Series	Weight
3819.346 } 3819.412 } ..	.247	+0.003	+0.006	+0.004	{ 1 { 1	A_1 } A_1 }	Low
3822.406 } 3822.470 } ..	.291	+ .001	+ .001	+ .002	{ 0 { 0	A_1 } A_1 }	Low
3830.745 } 3830.801 } ..	.636	− .001	+ .006	− .002	{ 0 { 0	B_1 } B_1 }	Low
3831.174 ...	.039	+ .005	+ .001	+ .002	3 d	A_1d	Low
3836.639 } 3836.689 } ..	.523	+ .007	+ .003	+ .002	{ 1 { 1	A_1 } A_1 }	Low
3844.378 ...	.239	+ .004	+ .001	+ .004	4 d	A_1d	Low
3846.131 ...	.997	− .002	+ .004	.000	2	B_1	High
3846.777 } 3846.814 } ..	.660	+ .004	+ .003	.000	{ 1 { 1	A_1 } A_1 }	Low
3851.427 ...	.295	+ .009	+ .003	+ .005	2 N	A_1d	Low
3852.541 ...	.406	+ .004	− .002	+ .002	2 N	A_1d	Low
3853.620 ...	.492	+ .001	+ .005	+ .004	2 N	A_1d	Low
3856.802 ...	.671	+ .006	+ .004	+ .007	2 N	A_1d	Low
3858.822 ...	.692	+ .008	.000	+ .002	2 N	A_1d	Low
3862.626 ...	.494	+ .005	+ .001	+ .001	2	A_1b	Low
3863.533 ...	.401	+ .002	− .001	.000	3 N	A_1b	Low
3864.438 ...	.308	+ .005	.000	+ .003	3	A_1	High
3865.282 ...	.154	+ .004	+ .004	+ .002	3	A_1sB_1	Low
3866.122 ...	.995	+0.002	+0.006	+0.004	3 N	A_1sB_2	Low
....	....	+0.0037	+0.0025	+0.0023	....	......	....

narrowest lines, the division between Sections A and B of Tables I and II has been based upon line-intensity. For this reason, and because of the relatively high precision of measurement, greater weight is to be attributed to the data of Sections A.

The series classification in the column next to the last in Tables I and II is from the work of Uhler and Patterson.[1] A_1 is the nominally "singlet" series from the first head, but in all probability it is a series of doublets. Its ultimate doublet character, according to Uhler and Patterson, appears at λ 3860.626, line 47 of the series,

[1] *Astrophysical Journal*, **42**, 434, 1915.

though preliminary widening is evident in line 44 at λ 3863.390. They say:

On leaving the head and going toward shorter wave-lengths one acquires the impression that the A_2 series (doublets from the first head) gradually grows out of and away from A_1. The first component of A_2 appears as a close companion of the eighth line of A_1. The second component of A_2 appears as a close component of line 13 of A_1 [λ 3881.587].

The number of lines suitable for measurement in the series A_1 is limited by the conditions that they be free from the A_2 lines and the disturbing influences of their own doublet character. In the shorter B_1 and C_1 series—singlets from the second and third heads—duplicity is not a disturbing factor, and the components of the A_2 and B_2 doublets are probably by nature single. The conditions that the corresponding solar lines, inherently more difficult to measure, should be measurable with fair accuracy, further limits the available material. Sections A of Tables I and II contain nearly all the lines under investigation that fulfil the conditions necessary for measurements of high weight.

DISCUSSION

The mean results from the data in Tables I and II are exhibited in Table III and may be briefly summarized as follows: The magnitude of the displacement to the red required by Einstein's deduction is approximately 0.008 A for the spectral region considered. Compared with this the observations at the center of the sun show a mean displacement of approximately 0.001 A to the violet for the 25 lines of greater weight, and of 0.001 A to the red for the 18 lines of lesser weight, or a zero displacement for the 43 lines. As a displacement due to relativity would be independent of the character of the radiating centers, this discrepancy between the calculated and observed values for the center of the solar disk requires some influence producing displacements of solar lines to the violet of just the magnitude to balance the predicted effect. Anomalous refraction, if active, would tend to displace the lines to the red; the wave-lengths of the lines employed are not appreciably affected by pressure; there is no evidence that wave-length depends upon the temperature of the source; of known causes there is left only the

possibility of an outward radial movement of the solar vapors with a velocity at this level of 0.634 km per second, which masks the assumed gravitational effect. At the sun's limb the radial movement would be across the line of sight, the Doppler effect would vanish, and the gravitational displacement of 0.008 A to the red should appear. For the lines whose measurement at the limb is most dependable, the 17 lines of Section A in Table II, the mean shift between the sun's limb and the arc is zero. There are three reasons for assigning a high weight to this mean, namely: under the

TABLE III

Comparative Solar and Terrestrial Wave-Lengths of Lines in the Nitrogen (Cyanogen) Bands

	Section A	Section B
At Center of Sun	25 Lines	18 Lines
Intensity	00–1	2–4
a) Direct comparison of solar and arc spectra	+0.0006	+0.0024
b) λ in center spectrum *minus* λ in arc spectrum	− .0002	+ .0012
c) (R−I) for band-lines *minus* "Standard" (R−I)	− .0030	+ .0010
d) (λ limb−λ arc) *minus* (λ limb−λ center)	− .0011	+ .0008
Mean (center−arc)	−0.0009	+0.0013
At Limb of Sun	17 Lines	18 Lines
Intensity	00–1	2–4
a) λ in limb spectrum *minus* λ in arc spectrum	0.0000	+0.0037
b) (λ center−λ arc)+(λ limb−λ center)	+0.0001	+0.0035
Mean (limb−arc)	0.0000	+0.0036

highest resolving power used the lines show no sign of duplicity; the narrowness of the lines reduces the chances for blends and in so far increases the probability that we are working with uncontaminated lines; and their isolation from adjacent lines is sufficient for measures of precision.

If the results of lower weight for the 18 compound and broader lines in Section B are included, the displacement to the red at the limb is 0.0018 A, or one-fourth of the calculated amount. The displacement shown by the broader lines may, however, be due to blends with lines of other elements. Adams found that, at the

limb, displacement of the metallic lines to the red is the general rule. In passing from the center to the limb the mean for 120 lines is 0.007 A. Undetected blends with such lines tend to introduce systematic displacement to the red at the limb for lines not sensitive to edge-effect and normally undisplaced.

Adams found for 14 lines of the ultra-violet fluting a shift, relative to the center, of 0.002 A to the red, with which the mean for the 35 lines in Table II is in agreement. He attributes this displacement between center and limb to rising convection currents. The 25 lines in Section A, Table I, lines of lowest level, indicate a rise of the vapors from the sun's interior of 0.08 km per second. This compares well with 0.12 km per second suggested by Adams. The displacement to the red in passing from the center to the limb for these low-level lines appears to be due to the disappearance at the limb of the Doppler-Fizeau effect in the ascending currents at the level where these weak lines originate. For the strong lines in Section B, the small displacement to the red at the sun's center may be accounted for by a downward drift at the higher level of origin, in which all high-level lines appear to be involved. For both the weak and the strong lines the displacement is, however, of the order of the errors in measurement, and great weight cannot be given to their distinctive behavior.

OTHER OBSERVATIONS

In an investigation bearing upon the behavior of the high-level calcium vapor in the solar atmosphere[1] the wave-lengths of the H and K lines were determined in the arc and at the solar limb, with results as follows:

	H_3	K_3
1 mm from the limb....	3968.478	3933.667
At the limb...........	3968.476	3933.665
In the arc.............	3968.476	3933.667

Though the close agreement between the solar and arc wave-lengths may in part be fortuitous and the evidence for equality

[1] St. John, *Mt. Wilson Contr.*, No. 48; *Astrophysical Journal*, **32**, 36, 1910.

should not be unduly stressed, the observations indicate with a high degree of probability that the difference is not of the order of 0.008 A, as required by the principle of relativity.

As the gravitational effect is proportional to wave-length, the displacement due to it should be more evident for lines of great wave-length. The Mount Wilson data on the comparative wave-lengths of the iron lines in the sun and arc give for 25 lines of the stable *b* group, mean λ 6300, a mean displacement to the red at the sun's center of 0.004 A instead of 0.013 A, required by the relativity theory, a discrepancy between calculated and observed values too large to be attributed to errors of observation.

A complete report of an investigation by Schwarzschild, "Über die Verschiebungen der Bande bei 3883 Å.E. im Sonnenspektrum,"[1] has not yet reached America. According to the abstract in the *Beiblätter*, he finds that at the center of the sun there is a displacement indicating a downward velocity of 0.2 km per second and that, if this downward movement is taken into account, there remain only slight displacements, which cannot be considered favorable to the Einstein theory.

For lack of fuller data a satisfactory comparison cannot be made between Schwarzschild's results and those in this paper. If the lines used by him are similar in character and intensity to those in Section B of Table I, as seems probable, there is good agreement between his displacement to the red interpreted as a downward movement of 0.2 km per second and the mean of 0.0014 A for the lines in Section B corresponding to a downward velocity of 0.1 km per second.

In *Bulletin* No. 39 of the Kodaikanal Observatory Evershed and Royds give the results of their measurement of 12 lines in the band at λ 3883 and their rather startling deduction of the receding of the solar atmosphere at the sun's center and all around the circumference where it is supposed to attain a velocity of recession of about 1 km per second. This suggested flow of the solar vapors from the hemisphere turned earthward at all points of the sun's limb is assumed to be due to a repulsion of the solar gases by the earth. They base this hypothesis upon a displacement to the red,

[1] *Beiblätter*, **39**, 480, 1915.

indicated by their measurement of lines in the band at λ 3883. They appreciate the difficulties involved in the idea, but consider that there is no alternative. The crucial point is the behavior of such lines at the sun's limb. As is evident from the summation in Table III, the Mount Wilson observations, based upon a larger number of lines, fail to confirm the results found at Kodaikanal. For the lines of highest weight there is no displacement to the red either at the center or at the limb. The measurements are inherently difficult, and results may be more or less influenced by the choice of lines and by the resolving power, definition, and dispersion of the spectrographs used.

The displacements between center and arc observed at Kodaikanal and at Mount Wilson, the appearance upon the Mount Wilson plates, and the estimated weights of the Mount Wilson measures are given in Table IV. The unresolved triplet groups (A_1, A_2, A_3), near the head of the band, where the A_2 doublets begin to appear and to separate from A_1, are unsymmetrical in the sun and arc. After the triplet character becomes discernible, the components vary progressively in intensity and spacing so that settings upon the complexes are difficult and liable to systematic errors until the separation is sufficient to free a line completely. The weights of the Mount Wilson measures are based largely upon the degree of isolation of the lines.

Evershed and Royds are of the opinion that the negative shifts for λ 3876 and λ 3877 are due to arc conditions and that the general shift to the red of 0.005 A observed by them is too small for the same reason. They state that the general effect of arc conditions is to reduce sun—arc shifts. When light is taken from near the poles of an iron arc or when a short arc with strong current is used, the sun—arc displacements for a large group of iron lines are negative, but this has not been observed for the lines of all elements. An examination of the carbon arc for pole-effect shows that the wavelengths of these band lines are free from this disturbing influence. Simultaneous exposures were made upon the pole and center of the carbon arc with the 30-foot spectrograph in the fifth order of a Michelson grating of 66,000 lines. The scale of the spectrograms is 3 mm to an angstrom. The result for lines of the series A_1, B_1,

C_1, A_2, and B_2 are given in Table V. There is no evidence of such instability as the iron lines of groups *d* and *e* show.

TABLE IV

THE APPEARANCE ON THE MOUNT WILSON PLATES OF THE LINES USED AT KODAIKANAL

Lines	Intensity	Center *minus* Arc		In the Arc	In the Sun
		Kodaikanal	Mt. Wilson		
3863.533....	3 N	+0.010	+0.002	A_1, broad, confused with B_2	Trace of line on red edge: weight very low in sun and arc
3864.438....	3	+ .008	+ .003	A_1	Free: weight high
3876.448....	0	− .002	− .003	A_1	Free: weight high
3876.556.... 3876.622....	0 0	+ .005	+ .003	A_2 A_2	Incomplete resolution: weight low
3877.481....	1	− .001	− .003	A_1	Free: weight high
3877.587.... 3877.646....	0 0	+ .006	+ .002	A_2 A_2	Red component > violet, weight low
3879.331.... 3879.394.... 3879.458....	1 0 0	+ .005	....	A_1 A_2 A_2	Not measurable as single line. A_2 blended with another line
3879.796.... 3879.851....	0 0	+ .003	+ .003	A_2 A_2	Partial resolution: weight low
3880.465.... 3880.532.... 3880.596....	1 2 1	+ .007	+ .003	A_1 A_2 A_2	Resolved triplet, middle component a blend, weight very low
3880.815.... 3880.931....	1 1	+ .004	+ .003	A_1 A_2 A_2	Resolved triplet, middle component weak, weight low
3881.729.... 3881.825....	1 1	+ .008	+ .002	A_1 first appearance of 2d component of A_2 series A_2 A_2	Apparent doublet, violet component of triplet blends with middle component, weight low
3882.828.... 3882.893....	1 0	+0.008	+0.002	A_1 first appearance of 1st component of A_2 series A_2	Unsymmetrical in arc and sun, weight low
Mean ...	....	+0.005	+0.0015	....	....

In the case of λ 3876 and λ 3877 an observed displacement to the violet might arise from the proximity of the lines on the red, if dispersion and resolution are insufficient, as there is then a tendency

on the part of the measurer to displace the violet and red components of close pairs to the violet and red, respectively.[1] On the Mount Wilson spectrograms the resolution is complete, and the separation from the adjacent lines is sufficient to show an equally intense background on both sides of the lines—conditions that experience has shown are necessary and sufficient for measures of precision. Two similar lines, λ 3879 and λ 3880, nearer the head and less separated from the adjacent lines on the red, whose measurement is presumably more liable to errors of this type, give 0.000

TABLE V

NEGATIVE POLE *minus* CENTER OF THE CARBON ARC

λ	Series	Pole-Center	λ	Series	Pole-Center
3819.197	B_1	0.000	3867.906	A_2	0.000
3825.256	C_1	− .001	3868.539	A_1	.000
3833.744	C_1	.000	3868.625	B_2	.000
3845.149	B_1	.000	3868.700	A_2	.000
3846.131	B_1	− .001	3868.785	A_2	.000
3864.438	A_1	.000	3872.312	A_2	+ .001
3867.118	A_2	.000	3876.448	A_1	.000
3867.205	A_2	−0.001	3877.481	A_1	0.000
			Mean		0.000

and +0.002 A, the mean of the four being −0.001 A. As a score of other lines of like intensity yield a similar result, it seems probable that small displacements are characteristic of these low-level lines and that within the limits of error displacements to the violet are not, on that ground alone, to be discarded. These observations remove the necessity felt by Evershed and Royds of assuming an effect of the earth driving the solar vapors from the sun's earthward-turned hemisphere and leave the question of limb-effect for further investigation, with pressure, level, and line-intensity as probable contributing factors.

I wish to express my appreciation of the assistance given in this difficult investigation by Miss Miller, who has had a large share in the measurements of each type, and to Miss Ware, who has checked the results of Miss Miller and myself in the most critical cases.

[1] St. John and Ware, *Mt. Wilson Contr.*, No. 120; *Astrophysical Journal*, **44**, 15, 1916.

SUMMARY AND CONCLUSIONS

1. The wave-lengths of a group of lines in the band spectrum of nitrogen (cyanogen) have been measured at the center and at the limb of the sun and in the carbon arc, in terms of identical iron standards and their solar wave-lengths have been redetermined in the Rowland system.

2. Emphasis was put upon the selection of lines, with a view to precision of measurement and freedom from blends, which is particularly important for observations of the limb, where measurements are inherently difficult and blends with metallic lines introduce systematic errors of the sign required by the relativity theory.

3. Sun−arc displacements have been determined at the center directly and by three indirect methods.

4. Limb−center and limb−arc displacements have each been determined by two methods.

5. The sun−arc displacements for iron lines in the region were measured, and from these a standard value for Rowland *minus* International was found for lines showing no displacement at the sun's center. With this the corresponding R−I for the selected band lines was compared.

6. The mean sun−arc displacement at the sun's center for 25 band lines of intensity 00–1 is −0.001 A, and for 18 lines of intensity 2–4 it is +0.0014 A, with a mean of zero for the 43 lines.

7. The mean sun−arc displacement at the limb for 17 band lines of intensity 00–1 is 0.000 A, and for 18 lines of intensity 2–4 it is +0.0036 A, with a mean of +0.0018 A for 35 lines.

8. The conclusion is that within the limits of error there is no evidence in these observations of a displacement to longer wave-length, either at the center or at the limb of the sun, of the order of 0.008 A, as required by the principle of relativity.

9. There is a limb-effect not due to motion, in which pressure, level, and line-intensity appear to be involved in varying degrees for different elements.

MOUNT WILSON SOLAR OBSERVATORY
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RELATION TO PROPER MOTION OF PREFERENTIAL MOTION AND OF THE PROGRESSIONS OF SPECTRAL CLASS AND MAGNITUDE-VELOCITY

By C. D. PERRINE

The following investigation concerns the relation between the preferential motion or streaming of the stars as interpreted by the ellipsoidal theory, and the size of proper motion, and is based upon radial velocities. It was suggested by the contrary behavior of 110 stars of magnitude 2.9 and brighter in the matter of the positions of the solar apex and by the consistently different apices derived for the A, F, and G stars from those given by the other spectral classes. These peculiarities correspond in general to differences in the sizes of proper motions.

It was desired also to see what effect there might be on the ellipsoidal motion of the A, F, and G stars by the use of the apex and velocity derived from these classes instead of the uniform apex and velocity used by Eddington and Hartley in their determination.[1] Advantage was taken, therefore, to combine both problems. The stars were separated according to brightness and also according to size of proper motion. The limit of annual proper motion was taken at 0".10. Those above this limit are called large and those below, small. The stars in Campbell's L. O. catalogues[2] were divided into two classes, those of 2.9 and brighter and those of 3.0 and fainter, for each spectral class. The mean magnitude of these latter is about $4\frac{1}{2}$, and they contain very few fainter than magnitude $5\frac{1}{2}$. Adams' Mount Wilson list[3] of 500 radial velocities was also available, the stars ranging almost entirely from 5.5 to 6.5 magnitude with a mean at about 6.0. This list contains no stars south of $-25\frac{1}{2}°$ declination and was selected especially with the view of extremes in the size of proper motion.

[1] *Monthly Notices*, **75**, 521, 1915.

[2] *Lick Observatory Bulletins*, **6**, 108, 1911; **7**, 20, 51, 113, 1913; *Publications of Lick Observatory*, **9**, 329, 1911.

[3] *Mt. Wilson Contr.*, No. 105; *Astrophysical Journal*, **42**, 172, 1915.

The limits adopted for the regions around the major and minor axes of the spheroid are practically the same as those used by Kapteyn and Adams[1] in similar investigations within 50° of either vertex for the major axes and from 60° to 120° from a vertex for the minor axes. The southern vertex used in the present investigation is that derived by Eddington[2] from the Boss stars, $\alpha = 274°$, $\delta = -12°$.

The results are given in the following tables, in which ρ_2 and ρ_1 are the major and minor axes respectively. The stars of Campbell's catalogues were cleared of solar motion as follows:

B	-20.0 km	toward	270°, +30°
A, F, G	$-18.$	toward	260°, +15°
K, M	-19.5	toward	270°, +30°

The solar motion used by Adams in obtaining the values of the velocities of the stars (v') given in his catalogue was -20 km toward 17^h59^m, $+30°.8$.

There is a consideration which at least ought to be mentioned in this connection. In the classification by size of observed proper motion the effect of the portion which is due to the sun's motion is neglected. It is not easy properly to eliminate this solar effect because of the lack of knowledge of distances. There are reasons for thinking that most of the conclusions resting upon such classifications are really free from any serious results of the unconscious selection which has thus been practiced. But until this matter has been put definitely beyond suspicion it seems well at least to bear in mind the possibility of some such effect.

The first value obtained for the ratio of the axes for the small μ of the F stars (Table I) was 1.91. An examination showed that the average μ of these stars was higher than the others. The experiment was then tried of rejecting the values over $0''.05$, with the result that the ratio was reduced to 1.17. These larger values were then included with the large μ. Two stars of 98 and 92 km were omitted in ρ_1 for large μ. None of the other classes were so treated with respect to restriction to smaller proper motions.

[1] *Proceedings Nat. Acad. Sci.*, 1, 14, 1915.

[2] *Stellar Movements*, p. 102.

These results in general show a smaller prolateness for the small than for the large proper motions. The fainter stars of K and M (of Adams' list, Table II), as well as the bright stars of the

TABLE I

PROLATENESS FROM RADIAL VELOCITIES

STARS OF 3.0 AND FAINTER FROM CAMPBELL'S L. O. CATALOGUES

Type	Small μ			Large μ		
	ρ_2	ρ_1	ρ_2/ρ_1	ρ_2	ρ_1	ρ_2/ρ_1
	km	km		km	km	
B......	(60) 7.9	(53) 8.2	0.96	....	....	......
A......	(64) 12.8	(63) 10.8	1.18	(8) 12.9	(23) 7.2	1.79
F......	(18) 12.2	(20) 10.4	1.17	(46) 19.4	(80) 11.9	1.63
G......	(47) 14.0	(34) 10.7	1.31	(13) 39.0	(28) 25.4	1.54
K......	(90) 15.3	(136) 14.1	1.09	(43) 29.1	(81) 20.5	1.42
M......	(18) 19.7	(34) 16.5	1.19	(4) 30.7	(9) 15.7	1.96

TABLE II

PROLATENESS FROM RADIAL VELOCITIES

ADAMS' MOUNT WILSON CATALOGUE OF 500 STARS

Type	Small μ			Large μ		
	ρ_2	ρ_1	ρ_2/ρ_1	ρ_2	ρ_1	ρ_2/ρ_1
	km	km		km	km	
B......	(52) 8.6	(29) 10.5	0.82	....	....	......
A......	(53) 11.3	(57) 9.2	1.23	(2) 19.4	(7) 10.3	1.88
	(49) 9.7	(55) 8.1	1.20	....	....	......
F......	(7) 12.8	(11) 9.6	1.33	(7) 51.4	(4) 25.7	2.00
	(6) 10.8	(10) 7.3	1.48	....	....	......
G......	(26) 12.8	(29) 7.8	1.64	(8) 79.4	(21) 27.6	2.88
	(23) 10.6	....	1.36	(7) 48.0	....	1.74
K+M...	(41) 14.3	(43) 12.2	1.17	(14) 30.4	(17) 25.5	1.19

TABLE III

PROLATENESS FROM RADIAL VELOCITIES

STARS 2.9 AND BRIGHTER FROM CAMPBELL'S L. O. CATALOGUES

Type	Small μ			Large μ		
	ρ_2	ρ_1	ρ_2/ρ_1	ρ_2	ρ_1	ρ_2/ρ_1
	km	km		km	km	
B......	(16) 8.9	(14) 4.9	1.81	....	....	......
A, F, G..	(9) 20.5	(9) 4.5	4.56	(8) 16.6	(17) 11.1	1.50
K, M...	(9) 11.2	(11) 9.1	1.23	(7) 16.7	(11) 19.1	0.87

same spectral classes, show no such difference. This may be only a coincidence, particularly in the case of the brighter stars, as the amount of data is very small, but requires further investigation from a larger number of stars. With the exception of the K and M stars already noted, the bright stars, including those of type B, show considerable values for the prolateness.

The evidence of the 30 bright B stars is quite consistent as far as it goes, as will be seen from the individual velocities which are given below.

Vertices	Minor Axis
+10.6	+8.5
24.3	+3.7
10.6	+6.3
9.3	+7.6
13.4	−9.0
4.9	−0.9
2.6	+5.3
1.0	+6.3
6.4	+5.3
7.6	+2.3
0.4	+7.6
2.8	+1.0
14.7	+8.0
10.1	+1.7
21.7	
+ 1.3	Mean 4.9
Mean 8.9	

Half of the velocities about the vertices are larger than any in the regions of the minor axis.

The increase of prolateness in the stars of large μ can be traced directly to the greater increase of velocity along the ellipsoidal axis. If we take the difference large μ minus small μ for ρ_2 and ρ_1, and then difference these in the sense ρ_2 minus ρ_1, we get the following:

LARGE μ *minus* SMALL μ, ρ_2 *minus* ρ_1

Class	Table I	Table II
A	+ 3.7 km	+ 7.0 km
F	+ 5.7	+22.5
G	+ 0.3	+15.4
K	+ 7.4	....
M	+11.8	....
K+M	....	+ 2.8
Mean	+ 5.8	+11.9

There is a dissymmetry or skewness in the average velocities with respect to the northern and southern groups about the minor axis which calls for remark in connection with the effect on the prolateness of the spheroid. It has been observed that in the classes A, F, and G the northern groups of the stars of small μ at right angles to the axis give smaller average velocities than the southern groups. In such cases there is a nearer equality of the southern group to the velocities from the vertices, the northern group standing out as a deficiency. Such a condition causes the minor axis to be reduced, and, when compared with the major axis, to show a prolateness. Indeed, if we take the average of the values of ρ_1 for the Campbell stars of $3^{M}_{.}0$ and fainter in the spectral classes A, F, and G (B not showing any prolateness), we find 7.8 for the northern groups and 11.6 for the southern. If we assume 11.6 for the major axis, we obtain a prolateness of 1.20 due entirely to such a deficiency in the velocity of the northern stars. If such stars had been unsymmetrically distributed and we had had no southern stars, we should have found a prolateness of 1.49 for these three classes, a value which differs but little from that obtained from the ellipsoidal theory. The small proper motions resulting for the A, F, and G stars of Adams' list give a mean value of 12.3 for ρ_2 and 8.9 for ρ_1 and are consistent in both cases. These values yield a prolateness of 1.38. Not yet having corresponding southern stars, it is not possible to judge with certainty, but this appears to be a case in point.

It is suggestive that we find such an effect only in these three classes where ellipsoidal motion is most pronounced. There is no marked effect of this sort in the B, K, and M stars ($3^{M}_{.}0$ and fainter, Campbell), where there is no prolateness for B, and values of only 1.09 for K and 1.19 for M stars. There are indications that this effect is in reality related to a dissymmetry on opposite sides (north and south) of the axis of solar motion for the stars of small μ. A well-marked effect of this kind which is believed to be an indication of rotation has been found in the B and K stars. The other classes have not yet been directly tested for this effect. This and other peculiarities of stars of both small and large μ will be made the subject of a separate investigation.

It is believed that these preferential motions are in the form of more or less definite streams, which it is hoped to isolate by means of their velocities.

The conclusion arrived at in this investigation from radial velocities, that the preference for motion along the ellipsoidal axis is almost entirely confined to the stars of larger proper motion (and presumably nearer), receives strong support from Dyson's[1] investigations of the larger proper motions, which showed (I infer from references, as I have not access to his original papers) a more pronounced streaming for these stars than for the groups used by other investigators where small proper motions were included. In their investigation of the relation between radial velocity and proper motion of the stars of classes F, G, K, and M, Kapteyn and Adams came to the conclusion[2] from much the same series of observations which I used that "the values of $\bar{\rho}_2/\bar{\rho}_1$ are no smaller for the stars having the very smallest proper motions than for the other stars in the list." Their table of results bears out this conclusion quite well. A minute examination, however, shows some slight signs of an increase of the prolateness with increasing size of proper motion. Combining the groups of the stars of smaller proper motion in the classes F and G, and separating the K and M stars at about 0."10, the following simple means of the ratios $\bar{\rho}_2/\bar{\rho}_1$ are obtained:

$\bar{\rho}_2/\bar{\rho}_1$

Class	Small μ	Large μ
F	1.51	1.93
G	1.52	1.99
K	1.17	1.33
M	1.40	1.61

This somewhat arbitrary and biased reclassification would have little weight by itself. The great differences between my results and those of Kapteyn and Adams are probably to be explained by the general deficiency in the radial velocities of the stars of small μ to the north of the ellipsoidal axis, combined with the fact that the

[1] *Proceedings Royal Society Edinburgh*, **28**, Part III, No. 13; **29**, Part IV, No. 21.

[2] *Proceedings Nat. Acad. Sci.*, **1**, 17, 1915.

Mount Wilson stars (which formed a not inconsiderable part of the investigation) are almost entirely northern stars, and would therefore have a tendency to increase the prolateness for the groups of smaller μ.

RELATION OF PROPER MOTION AND RADIAL VELOCITY TO DISTANCE

Kapteyn and Adams[1] showed that there is an increase of radial velocity with increasing proper motion in the spectral classes F, G, K, and M, and they were able to satisfy this condition by a non-Maxwellian distribution, with indications also of a relation to magnitude, without the aid of a third explanation of the decrease of velocity with increasing distance. The latter is, perhaps, the most direct interpretation of such an effect as that observed, and deserves consideration in view of the conclusion indicated by the present investigation that the difference in the behavior of the stars of large and small proper motion is primarily due to distance.

It is inconceivable with even the little absolute knowledge which we have of the distances of the stars that, in general, the brighter stars and those with the largest proper motions should not be nearest. The direct determinations of the parallactic displacements of a number of the stars of large proper motion confirm this explanation in a way which seems final. There are as yet only a small number of well-determined parallaxes, but these, as far as they go, indicate greater radial velocity for the nearer than for the more distant stars. Forty-one stars have been found whose parallaxes are $0\rlap{.}''06$ and over, which appear to be reliable—the probable error being well below the foregoing value. These were arranged in three groups, as given in Table IV. The second group contains a considerable number of large velocities. If we reject four with velocities of from 73 to 98 km, we obtain the results given in the second line of the second group in Table IV.

If these rejections are justifiable, the results show a definite decrease of velocity with increasing distance. This is shown also in another and probably more reliable way by this same group of 41 near stars. The mean velocity of the group is 26.9 km (or

[1] *Proceedings Nat. Acad. Sci.*, 1, 14, 1915.

20.7 km, if we omit the four stars with velocities of from 73 km to 98 km), whereas the average velocity of a large number of stars of similar magnitude and spectral class (including many stars of small μ), which are undoubtedly at a much greater distance than the 41 stars, may be taken at 15 km or 10 km, if limited to small proper motions. The 41 stars are very well distributed with respect to the different parts of the ellipsoid, and appear to be comparable on the whole with those giving the smaller velocities in all ways except distance.

TABLE IV

RELATION OF RADIAL VELOCITY TO PARALLAX

π	Mean Mag.	Mean π	Mean v	No. Stars	Mean μ
			km		
0".06 to 0".09	3.1	0".07	17.9	13	0".63
0.10 to 0.19	{4.5	0.13	34.1	18	1.73
	{4.3	0.13	19.8	14	1.17
0.20 to 0.76	2.9	0.34	25.5	10	2.22

Such conclusions presuppose that the Doppler effect is not sensibly modified by distance.

The number of stars is small and the weight of the evidence, therefore, not great. As far as it goes, however, it is confirmatory of the simple explanation that the radial velocities (and perhaps the total velocities) of the nearer stars are, on the average, greater than those of the more distant stars. The conclusion that the difference in velocity is due to distance appears to be strengthened by the investigations of Kapteyn and Adams, already referred to, in which they find (p. 18) that "a peculiar feature of this result is that if all the stars are used, and not alone those on which the stream motion has little influence, the exponential constants remain essentially unchanged." In other words, I understand this to mean that the observed non-Maxwellian distribution has no relation to the preference for motion along the ellipsoidal axis, that the distribution observed is more general than that. The factor of distance would seem to supply such a general requirement. It would be of interest to know how these constants would be affected by limitations in the matter of proper motions.

THE PROGRESSION OF RADIAL VELOCITY WITH SPECTRAL CLASS

Evidence has been encountered in this work that the spectral class velocity-progression may also depend upon the proper motions (or total motions), that the radial velocities of the stars of small proper motion and presumably great distance are on the whole nearly free from this effect. Of Adams' list 326 of the stars fail to show any progression whatever with spectral class when classified with respect to the ellipsoidal axis, and a number of groups of small proper motion of Campbell's catalogues also fail to show such progression when similarly classified. Table V shows the results

TABLE V

STARS OF SMALL μ

ADAMS' MOUNT WILSON CATALOGUE

Spectral Class	All Stars		Rejecting R.V. of 30 km or Over		
	ρ_2	ρ_1	ρ_2	ρ_1	Mean ρ
	km	km	km	km	
B..........	(52) 8.6	(29) 10.5	....	....	9.6
A..........	(53) 11.3	(57) 9.2	(49) 9.7	(55) 8.1	8.9
F..........	(7) 12.8	(11) 9.6	(6) 10.8	(10) 7.3	9.0
G..........	(26) 12.8	(29) 7.8	(23) 10.6	(29) 7.8	9.2
K..........	(28) 14.0	(26) 10.1	(25) 11.6	(24) 8.1	9.6
M..........	(13) 14.8	(17) 15.4	(11) 10.0	(13) 11.9	10.9

from all the stars of small proper motion in Adams' catalogue as originally selected and after the rejection of 22 of these stars having velocities of 30 km or over. All of the 348 stars show a slight increase of velocity with spectral class. After the rejection of the 22 stars there is no increase whatever, which indicates, in these stars at least, that the progression with spectral class has been due to the 22 stars with large velocities. The proportions of the stars of very small μ but relatively large radial velocity of these stars of Adams' list are found to be progressive, the later spectral classes containing larger proportions than the earlier classes. The distribution of such stars in the sky appears to be somewhat systematic. Interpreted literally, such progression would seem to indicate a tendency among the later types for the motions to become radial.

It should be recalled in this connection that a small proportion of stars will be moving nearly in the line of sight, and therefore, if selected on a basis of proper motion alone, that a few will show abnormally large radial velocities. It would seem to be justifiable to classify such stars of large radial motion upon the general ground of total motion.

It is of interest in this connection to classify with respect to magnitude, and northern and southern regions of sky, the values of ρ_1 for the stars of small μ in the catalogues of both Campbell and Adams. These are given in Table VI. As previously used, the F stars were restricted to those having proper motions of $0''.05$ or less.

TABLE VI

ρ_1 FOR STARS OF SMALL μ

CAMPBELL'S L. O. CATALOGUES. NORTHERN AND SOUTHERN SEPARATELY

Spectral Class	$2^{M}.9$ and Brighter	$3^{M}.0$ and Fainter	Spectral Class	$2^{M}.9$ and Brighter	$3^{M}.0$ and Fainter
	km	km		km	km
Northern			Southern		
B.........	(3) 6.2	(16) 7.8	B.......	(11) 5.0	(37) 8.4
A.........	(3) 4.2	(33) 8.1	A.......	0 ...	(30) 13.8 (27)† 11.7
F.........	(1) 7.3	(4)* 5.5	F.......	(2) 6.0	(14) 10.2
G.........	(1) 1.8	(13) 8.2	G.......	(2) 3.2	(21)‡ 12.3 (19)† 8.2
K.........	(5) 10.9 (4) 6.2	(54) 14.4 (47)† 10.9	K.......	(5) 8.8 (4) 6.3	(82) 13.9 (73) 10.8
M.........	0	(12) 16.7 (9) 8.3	M.......	(1) 1.5	(22) 16.4 (19) 12.2

* One star of 39 km omitted.
† Omitting velocities of 30 km and over.
‡ One star of 227 km omitted.

The 34 stars of magnitude 2.9 and brighter according to this classification show no certain increase of velocity with spectral class. The slightly larger values of the K stars indicate at first sight a small increase, but if we reject one star out of each series (29.7 and 19.0) the remainder show no real increase.

The northern stars of $3^{M}0$ and fainter show a sudden break between the G and K types, whereas in the southern stars a similar break occurs between B and A. If in these 338 stars we reject the 27 stars with velocities of 30 km and over (8 per cent), there is

little or no evidence in the remaining 311 of an increase with spectral type. Further restriction in the size of proper motion would probably have some effect also. The number of stars is small, and it seems better to await more material before making a further discussion of the matter. Table VII contains the values

TABLE VII

ρ_2 FOR STARS OF SMALL μ

CAMPBELL'S L. O. CATALOGUES

Type	$2^{\mathrm{M}}.9$ and Brighter	$3^{\mathrm{M}}.0$ and Fainter	Type	$2^{\mathrm{M}}.9$ and Brighter	$2^{\mathrm{M}}.0$ and Fainter
	km	km		km	km
B...........	(16) 8.9	(60) 7.9	G..........	0	(47) 14.0 (44) 11.4
A...........	(4) 25.6	(64) 12.8 (56) 10.5	K..........	(6) 8.8	(90) 15.3 (80) 12.5
F...........	(5) 16.4 (4) 12.3	(18) 12.2	M..........	(3) 16.0 (2) 6.1	(18) 19.7 (13) 12.4

for ρ_2 from Campbell's catalogues in the two classes of magnitudes previously adopted. If large radial velocities are rejected from a number of these groups on the hypothesis previously adopted, there is left no real progression with spectral class. No rejections were made in the stars of class A of $2^{\mathrm{M}}9$ and brighter because at least three of the four belong to the class of larger motion.

If we accept the evidence, which seems direct and strong, that the nearer stars are on the whole moving with greater velocities, in the line of sight at least, than the distant ones, then the greater part of the observed increase of velocity with spectral type appears to be very well explained upon the simple assumption of a greater proportion of nearer stars in the later spectral types. Adams came to the conclusion[1] that a direct interpretation of his results indicated that among the very distant stars the change of velocity with spectral type is slight. It seems to me probable that the slight effect still remaining in his results is entirely similar to that which I found to result from a very few comparatively large velocities in the later types among the stars of small proper motion, the bearing of which has been considered to some extent above.

[1] *Mt. Wilson Contr.*, No. 105, p. 21; *Astrophysical Journal*, **42**, 172, 1915.

It may be noted in this connection that if there is a loss of light in the stars as they progress in spectral type from B to M (as seems very probable) there would be a tendency, in any classification according to a given brightness, for the later types to be nearer on the average. If, then, the conclusion arrived at earlier, that the nearer stars have larger radial velocities and proper motions than the more distant ones, is correct, there will be an apparent increase of radial velocity with spectral class in stars so classified. Such a suggestion was indeed made by Eddington, but later was considered to be entirely disproved.[1]

The fact that there appears to be a close relation between proper motions and radial velocities would seem to indicate that the increase of motion with decreasing distance is in three dimensions.

THE MAGNITUDE-VELOCITY EQUATION

It is of some importance to see what effect this classification with respect to proper motion (and the ellipsoidal axis) will have upon the observed increase of radial velocity with decrease in brightness. The values of ρ_2 and ρ_1 from the stars of small and large μ for all types for the three classes of brightness are given in Tables VIII and IX. ρ_2 and ρ_1 have the same meaning as before. μ is the angular motion on a great circle and ρ_2' and ρ_1' are the axes reduced to unity (ρ_2/μ and ρ_1/μ). The numbers of stars in each group are given in parentheses. The small numbers of stars of 2.9 and brighter of large μ are greatly affected by a few very large motions, as will be seen by the effect of omitting the two largest in each group.

The stars of small μ show no change of velocity with change of brightness. The stars of large μ show a well-defined increase of velocity with decreasing brightness, greatest in the direction of the major axis. When, however, the velocities for the different groups are reduced to a common proper motion, this progression disappears almost completely. The slightly smaller values for the stars of 2.9 and brighter which remain can scarcely be held to furnish any real evidence of an effect due to magnitude. However, on account

[1] *Stellar Movements*, p. 161.

of the very consistent progressions which have been shown in so many other classifications, and until the condition that the nearer bright stars also approximate more closely to the sun in spectral type is satisfactorily accounted for, the conclusion that such progressions have alone resulted from varying mixtures of stars of different distances in relation to ellipsoidal motion will require further careful study.

TABLE VIII

CLASSIFICATION ACCORDING TO BRIGHTNESS

SMALL μ

	ρ_2	ρ_1	ρ_2/ρ_1
	km	km	
$2^M.9$ and brighter	(34) 13.5	(34) 6.2	2.18
3.0 to $5\frac{1}{2}$	(297) 13.6	(317) 11.8	1.15
$5\frac{1}{2}$ to $6\frac{1}{2}$	(179) 12.4	(169) 10.4	1.19

TABLE IX

CLASSIFICATION ACCORDING TO BRIGHTNESS

LARGE μ

	ρ_2	μ	ρ_2'	ρ_1	μ	ρ_1'	ρ_2/ρ_1	ρ_2'/ρ_1'
	km		km	km		km		
$2^M.9$ and brighter	(15) 16.6	0.″42	39.5	(28) 13.8	0.″62	23.1	1.28	1.71
	(13) 15.9	0.29	54.8	(26) 13.6	0.38	35.8	1.17	1.53
3.0 to $5\frac{1}{2}$	(96) 26.4	0.38	69.3	(212) 17.6	0.44	40.2	1.57	1.72
$5\frac{1}{2}$ to $6\frac{1}{2}$	(31) 45.2	0.64	70.6	(49) 22.3	0.56	39.8	2.03	1.77

The tentative suggestion of B. Boss,[1] "it would appear that within this region (a space around the sun having a radius equal to the distance represented by $M=6''$ [$\pi=0.''015\pm$]) the evolution of stars has been going on so long that almost all the B stars that may have once existed there had either moved out or had developed into older types," may have a bearing on this point. A difficulty for the first of these explanations would arise, however, if in reality the velocities depend simply upon distance or, what seems more probable, on their position in a system which has different velocities in different parts. *Such a condition would not provide any mechan-*

[1] *Astronomical Journal*, **26**, 189, 1911.

ism by which a sorting-out or mixing process could take place, such as would result from increases of velocity with age or decreasing brightness. Under such circumstances, in order to account for the tendency to concentration of the intermediate spectral types (with larger motions) nearer the sun, it would seem necessary to suppose that these nearer stars form a group which has had a separate origin or different development from the rest of the system, as indicated by Boss in the second of his suggestions above.

Such an assumption destroys at once the idea of a common origin in time and space for all the stars of our system. Whether such a hypothesis is tenable remains to be seen. If the stars have in reality evolved from the great irregular nebulae (of which we see some signs), such a condition would appear to favor such a theory of separate origins for groups of stars in the same system, in place of a single common history.

So long as the increase of velocity with proper motion was supposed to have no relation to distribution in space, but to be purely accidental in that respect, some mechanism of acceleration which would account for such differences of velocity as have been so abundantly observed was necessary. If, however, the relation of radial velocity to proper motion is in fact a relation to distance, it is almost certainly connected with some general motion of the system, and the necessity for accelerations is much less or perhaps non-existent. Motions which would very well satisfy the conditions as this investigation shows them might be conceived to exist in the spiral nebulae, for example, where the stream velocities near the center would be in very different directions from those in the outer portions of the arms, and where the actual velocities may differ widely also.

The suspicion exists that behind this apparently simple dependence of preferential motion upon proper motion or distance lurks another cause which is related to, or is, the real general motion (or motions) of the system. I have encountered in practically every considerable group of stars which I have examined and discussed in other ways, particularly in the radial velocities, a tendency among the residuals *not* to follow the law of accidental errors even approximately. That this is but another manifestation of the effects of

Kapteyn's two streams seems very probable. These peculiarities are being made the subject of a separate investigation.

Discussion of many questions and relations which suggest themselves, including the bearing upon the general motions of the stellar system, is reserved, awaiting more complete confirmation and the conclusion of other investigations, particularly of the stars of small μ. Although these latter appear to be devoid of preference for the ellipsoidal axis, they give some indications of a different form of preferential motion.

CONCLUSIONS

I. The preference for motion in the direction of the ellipsoidal axis appears to be confined to the stars of larger proper motion and brighter than 3.0 magnitude in all of the spectral classes, with the possible exception of K and M. The limit of proper motion below which little or no preference for the ellipsoidal axis is shown appears to be about 0".05.

II. The increase in prolateness in the stars of large proper motion can be traced chiefly to the direction of the ellipsoidal axis.

III. This relation to proper motion furnishes a satisfactory explanation of the peculiar behavior of the stars of class B.

IV. The radial velocities of the nearer stars are larger than those of the more distant stars.

V. When classified according to size of proper motion, and when a few large motions are excluded, there is no certain increase in the radial velocities of the different spectral classes.

VI. When the radial velocities are classified according to size of proper motion, there is no certain change with magnitude.

Observatorio Nacional Argentino, Córdoba
March 31, 1917

NOTES ON THE COLOR-CURVE AND LIGHT-ELEMENTS OF W URSAE MAJORIS[1]

By HARLOW SHAPLEY AND J. VAN DER BILT

The variable W Ursae Majoris[2] is of particular interest because of its exceptionally short period (four hours between minima) and because when interpreted as an eclipsing binary its mean density is more than twice that of the sun. Discussions of the orbit have been published by H. N. Russell[3] and H. Shapley.[4] A further study of the light-variations and orbit, on the basis of simultaneous photographic and photo-visual observations, was begun with the 60-inch reflector in 1916. The work was discontinued, however, upon learning that a similar investigation, based on Potsdam visual observations and unpublished photographic measures from Harvard, was already under way at Princeton.[5] The following pages contain a brief discussion of the color-curve obtained from the preliminary Mount Wilson observations, and an investigation of the light-elements based upon a considerable number of published and unpublished observations of minima during the last thirteen years.

OBSERVATIONS AT MOUNT WILSON AND UTRECHT

The magnitudes of the comparison stars used on the photographs with the 60-inch reflector were determined in the usual manner from ten polar-comparison plates (Table I). Magnitudes of the variable, derived from a series of Seed 27 and Cramer isochromatic plates made on April 26, 1916, are given in Table II. Six, seven, or eight exposures were made on each plate with exposure-

[1] *Contributions from the Mount Wilson Solar Observatory*, No. 140.

[2] W Ursae Majoris = B.D. $+56^\circ 1400$; $\alpha = 9^h 36^m 43^s$, $\delta = +56^\circ 24'.5$ (1900.0); spectrum G.

[3] *Astrophysical Journal*, **36**, 139, 1912.

[4] *Contributions from the Princeton University Observatory*, No. 3, p. 82, 1915.

[5] Since this paper was written the Princeton results have appeared in *Astrophysical Journal*, **45**, 306, 1917.

times of one-half, one, or two minutes. The phases were computed with the formula given below. A plot of these results shows that the color-index on the Mount Wilson system is about +0.85, with little evidence of change during the cycle of light-variation.[1]

TABLE I

MAGNITUDES OF COMPARISON STARS

STAR	B.D.	*H.A.*, 63 VISUAL	MOUNT WILSON		COLOR-INDEX
			Photo-visual	Photographic	
a...........	+56°1397	....	6.64	7.65	+1.01
b...........	+56 1399	8.74	8.76	9.90	+1.14
c...........	+56 1398	8.73	8.80	10.08	+1.28

TABLE II

MOUNT WILSON OBSERVATIONS OF W URSAE MAJORIS
April 26, 1916; J.D. 2420980

PHOTOGRAPHIC					PHOTO-VISUAL				
Plate	Gr. H.M.T.	Apertures	Phase	Mag.	Plate	Gr. H.M.T.	Apertures	Phase	Mag.
3034P .	^{d}672	14, 9, 6	−0^{d}142	8.76	3035P	^{d}676	14, 9, 6	−0^{d}138	7.91
3036...	.702	14, 9	− .112	8.70	3037..	.706	14	− .108	......
3038...	.720	14, 9	− .094	8.64	3039..	.724	14, 9	− .090	7.88
3040...	.737	14, 9	− .077	8.81	3041..	.740	14, 9	− .074	7.96
3042...	.754	14, 6	− .060	8.84	3043..	.758	14, 9	− .056	8.07
3044...	.771	14, 9	− .043	8.96	3045..	.774	14, 9	− .040	8.03
3046...	.789	14	− .025	9.06	3047..	.792	14, 9	− .022	8.18
3048...	.802	14, 9, 6	− .012	9.20	3049..	.805	14, 9	− .009	8.41
3050...	.814	14, 9	.000	9.27	3051..	.817	14, 9	+ .003	8.29
3052...	.829	14, 9, 6	+ .015	8.79	3053..	.832	14, 9	+ .018	8.15
3054...	.841	14, 9	+ .027	8.67	3055..	.844	14, 9	+ .030	7.98
3056...	.856	14, 9, 6	+0.042	8.52	3057..	.860	14, 9, 6	+0.046	7.81

Though the light-curves of this variable often show an asymmetry which, if real, could not be accounted for completely by the simple eclipse theory, the star evidently is not a Cepheid variable, as is sometimes supposed, since no conspicuous change of color is found. Moreover, Russell has no difficulty in interpreting the light-curve, or in explaining the differences between visual and photographic curves through differences in darkening at the limb.[2]

[1] The last two plates are uncertain because of the low altitude of the field.

[2] *Popular Astronomy*, **25**, 30, 1917.

At the Utrecht Observatory 105 observations of W Ursae Majoris were obtained with the polarizing photometer attached to the $4\frac{1}{2}$-inch refractor.[1] At maximum the variable is a little too bright for this instrument, but the minima can be determined very sharply. The following comparison stars were used:

a=B.D.+54°1329=Potsdam Gen. Cat. 5731;		Mag.=7.73
b=	+56°1412=a+0^{M}.33	8.06
c=	+56°1399=a+1 38	9.11

The brightness of both b and c was derived from the measured differences with a.

The method of observation differed somewhat from the usual one inasmuch as more settings were made on the rapidly changing variable than on the comparison stars. In the interval covered by the observations on each evening three to five measures of each of the comparison stars were secured, and these were plotted against the time on squared paper. After allowance for extinction, the settings were usually remarkably constant and could be read accurately for the times corresponding to the measures on the variable. In this way the latter could be closely followed through the steep ascents and descents of its light-curve.

The Utrecht observations are given in detail in Table III, the fifth column containing the differences in hundredths of a magnitude between the adopted value and that given by each of the comparison stars.

THE LIGHT-ELEMENTS

Since about 6000 epochs have elapsed since the latest discussion, a revision of the light-elements can now be made. In view of the possibility that the light-curve may be changeable in form, as many individual minima are included in the computation as can be derived with sufficient accuracy from the published series of observations.

The list in Table IV contains 67 minima, extending over an interval of 29,273 epochs. That numbered 61 has not been used in the derivation of elements. Nos. 59 and 60 were received from Professor Jordan after the completion of the solution and have been

[1] The instrument has been described in *Recherches astronomique de l'Observatoire d'Utrecht*, **6**, 10, 1916.

TABLE III

Utrecht Observations of W Ursae Majoris

Date	J.D. and Gr.M.T. 2420000+	Phase	Mag.	Residuals* a	b	c
1916 April 28	0982.461	0ᵈ071	8.42	+5,	....,	−5
	.505	.116	8.16	−2,	....,	+3
	.548	.158	7.72	+1,	....,	0
April 29	0983.408	.018	8.07	−9,	....,	+9
	.420	.029	8.11	−7,	....,	+7
	.429	.039	8.06	−5,	....,	+5
	.440	.049	8.14	−1,	....,	+2
	.452	.062	8.24	0,	....,	0
	.503	.113	8.30	+1,	....,	0
	.515	.125	8.23	+2,	....,	−2
	.524	.134	7.97	+3,	....,	−2
	.535	.144	7.90	+4,	....,	−3
	.544	.153	7.87	+5,	....,	−5
May 11	0995.408	.008	7.80	−11,	....,	+12
	.421	.021	7.94	−9,	....,	+10
	.428	.028	7.87	−8,	....,	+8
	.433	.033	7.92	−7,	....,	+7
	.438	.038	8.02	−6,	....,	+7
	.444	.044	8.06	−5,	....,	+6
	.450	.050	8.15	−5,	....,	+5
	.459	.059	8.24	−4,	....,	+5
	.468	.068	8.48	−4,	....,	+5
	.473	.073	8.43	−4,	....,	+4
	.480	.080	8.65	−4,	....,	+5
	.489	.089	9.09	−3,	....,	+3
	.495	.095	8.71	−1,	....,	+2
	.501	.101	8.76	0,	....,	+1
	.508	.108	8.54	+1,	....,	0
	.514	.114	8.30	+1,	....,	0
	.525	.125	8.16	+1,	....,	0
	.547	.147	7.99	+2,	....,	−1
	.556	.156	7.99	+1,	....,	0
May 17	1001.410	.005	7.92	....,	+1,	−1
	.423	.018	7.92	....,	+1,	0
	.428	.023	7.88	....,	0,	0
	.434	.029	7.97	....,	0,	0
	.438	.033	7.99	....,	0,	0
	.448	.043	7.97	....,	−2,	+3
	.461	.056	8.14	....,	−3,	+3
	.465	.060	8.19	....,	−3,	+3
	.469	.065	8.28	....,	−3,	+4
	.474	.069	8.39	....,	−4,	+5
	.483	.078	8.48	....,	−5,	+5
	.486	.082	8.60	....,	−5,	+5
	.491	.086	8.69	....,	−5,	+5
	.495	.090	8.70	....,	−5,	+5
	.500	.095	8.55	....,	−5,	+6
	.505	.101	8.44	....,	−5,	+5
	.517	.112	8.32	....,	−5,	+6
	.527	.122	8.02	....,	−6,	+6
	.536	0.131	7.89	....,	−5,	+6

*A positive residual means that the adopted magnitude is brighter than that derived from the omparison star.

TABLE III—*Continued*

Date	J.D. and Gr.M.T. 2420000+	Phase	Mag.	Residuals *a*	*b*	*c*
1916	1003.408	$0^{d}001$	7.95	+ 8,	− 4,	− 3
	.415	.009	7.88	+ 5,	− 3,	− 3
	.424	.019	7.98	+ 3,	− 2,	− 2
	.429	.024	7.96	+ 1,	− 1,	− 1
	.434	.027	8.00	0,	+ 1,	− 1
	.439	.034	7.99	− 2,	+ 2,	0
	.448	.043	8.01	− 4,	+ 3,	0
	.456	.051	7.99	− 4,	+ 4,	+ 1
	.465	.060	8.18	− 4,	+ 3,	+ 2
	.469	.064	8.26	− 4,	+ 3,	+ 2
May 19..........	.473	.068	8.50	− 4,	+ 2,	+ 3
	.478	.072	8.53	− 4,	+ 2,	+ 3
	.482	.076	8.41	− 5,	+ 1,	+ 3
	.487	.081	8.47	− 5,	0,	+ 4
	.496	.090	8.70	− 3,	− 1,	+ 4
	.500	.094	8.71	− 3,	− 1,	+ 3
	.505	.098	8.65	− 2,	− 1,	+ 3
	.510	.104	8.41	− 2,	− 1,	+ 2
	.520	.113	8.26	0,	0,	+ 1
	.530	.123	8.05	− 1,	+ 2,	− 2
	.541	.135	7.93	− 3,	+ 4,	0
	.554	.148	7.89	− 6,	+ 5,	+ 1
	1005.415	.007	8.18	....,	+ 2,	− 2
	.421	.013	8.03	....,	+ 3,	− 3
	.433	.025	8.10	....,	+ 6,	− 6
	.443	.035	7.90	+ 9,	+ 1,	−10
	.451	.043	8.01	+ 7,	+ 3,	− 9
	.459	.051	7.97	+ 6,	+ 2,	− 7
	.466	.058	8.14	+ 5,	0,	− 6
	.472	.064	8.30	+ 4,	0,	− 5
	.475	.067	8.35	+ 5,	0,	− 5
May 21..........	.479	.071	8.44	+ 5,	− 1,	− 4
	.483	.075	8.60	+ 5,	− 1,	− 4
	.487	.079	8.63	+ 5,	− 2,	− 3
	.490	.082	8.54	+ 5,	− 2,	− 2
	.496	.088	8.84	+ 5,	− 2,	− 4
	.500	.092	8.86	+ 6,	− 1,	− 5
	.505	.097	8.70	+ 7,	0,	− 6
	.509	.101	8.62	+ 8,	0,	− 8
	.516	.108	8.51	+10,	+ 1,	−11
	.526	.118	8.28	+10,	+ 2,	−13
	.533	.125	8.09	+11,	+ 2,	−14
	1012.420	.007	8.08	....,	+ 3,	− 2
	.432	.018	8.01	....,	0,	0
	.437	.023	7.96	....,	− 1,	+ 2
	.442	.028	7.93	....,	− 3,	+ 3
	.450	.036	8.07	....,	− 6,	+ 6
May 28..........	.459	.045	8.26	....,	− 6,	+ 6
	.480	.067	8.52	....,	− 2,	+ 2
	.486	.073	8.44	....,	0,	0
	.499	.085	8.74	....,	0,	+ 1
	.509	.095	8.57	....,	− 2,	+ 2
	.517	.104	8.54	....,	− 1,	+ 2
	.525	0.112	8.34	....,	+ 1,	0

TABLE IV

LIST OF MINIMA

No.	Observed Minima Date		Gr.M.T.	Red. to Sun	Epoch	Obsd. J.D. (Helioc.)	O.–C. Days	O.–C. Min.	Observer	Remarks	Reference
1	1903	Jan. 14	$4^h 34^m$	$+6^m$	0	6129.194	−0.006	8.0	MK	....	Ap. J., **17**, 205
2		17	4 40	+6	18	6132.199	− 4	5.2	"	....	
3		17	8 31	+6	19	6132.359	− 11	15.3	"	....	
4		18	8 34	+6	25	6133.361	− 10	13.8	"	....	
5		Feb. 20	5 25	+6	222	6166.230	− 4	5.2	"	....	A.N., 4005
6		Apr. 24	10 51	0	601	6229.452	− 7	9.5	"	....	
7	1904	Jan. 10	8 50	+6	2165	6490.372	+ 7	10.7	F	....	A.N., 3963
8		17	8 59	+6	2207	6497.378	+ 7	10.7	"	....	
9		27	5 17	+6	2266	6507.224	+ 11	16.4	"	Uncertain	
10		27	9 12	+6	2267	6507.387	+ 7	10.7	"	Good	
11		Feb. 13	5 14	+6	2368	6524.222	− 7	9.5	MK	....	A.N., 4005
12		17	9 40	+6	2393	6528.407	+ 8	12.1	F	Fair	A.N., 3963
13		19	5 43	+6	2404	6530.242	+ 8	12.1	"	Fair	
14		Mar. 14	6 30	+4	2548	6554.274:	+ 18:	26.5	"	Uncertain	
15	1905	Jan. 17	8 43	+6	4401	6863.367	− 6	8.0	MK	....	A.N., 4005
16		Nov. 9	19 15	+1	6178	7159.803	− 8	11.8	PJ	Fair	Ap.J., **23**, 84
17		Dec. 7	20 7	+4	6346	7187.841	+ 4	5.9	"	Poor	
18	1906	July 26	9 7	−6	7728	7418.376	− 6	8.0	MK	Good	A.N., 4128
19	1907	Mar. 4	5 59	+5	9052	7639.253	+ 2	3.3	T	Orange filter*	*Pulkowa Mitt.*, **2**, No. 21
20		5	10 0	+5	9059	7640.419	+ 1	0.9	"	Green-blue†	
21		7	9 55	+5	9071	7642.417	− 3	4.9	"	Orange‡	
22		24	6 24	+3	9172	7659.269	− 0	0.1	"	Orange*	
23		29	6 36	+3	9202	7664.277	+ 3	4.9	"	Ultra-violet*	
24		30	6 40	+3	9208	7665.280	+ 3	4.9	"	Ultra-violet*	
25		Apr. 1	6 42	+2	9220	7667.281	+ 5	6.6	"	Ultra-violet‡	
26		10	6 54	+2	9274	7676.289	+ 4	6.2	"	Ultra-violet*	
27		12	6 54	+1	9286	7678.288	+ 2	2.2	"	Orange§	
28		13	6 59	+1	9292	7679.292	+ 5	6.6	"	Green-blue‡	
29		15	7 1	+1	9304	7681.293	+ 4	5.5	"	Greer-blue†	
30		29	11 11	0	9389	7695.466	− 3	4.2	"	Orange*	
31		May 1	11 23	0	9401	7697.474	+0.003	4.8	"	Ultra-violet*	

32	1907	June 8	8^h 9^m	-4^m	9628	7735.337	−0.002	2.3	Ba	Fair	M.N., **69**, 86
33		9	8 14	−4	9634	7736.340	+ 0	0.6	"	Good	
34		19	8 24	−5	9694	7746.347	− 2	2.3	"	Fair	
35		July 18	9 7	−6	9868	7775.376	+ 0	0.6	"	Good	
36		22	9 7	−6	9892	7779.376	− 4	5.2	"	Good	
37		25	9 10	−6	9910	7782.378	− 4	5.2	"	Good	
38		Aug. 6	9 23	−6	9982	7794.387	− 6	8.0	"	Fair	
39		11	9 38	−6	10012	7799.397	− 1	0.9	"	Uncertain	
40	1908	Jan. 2	8 45	+6	10875	7943.369	+ 6	9.2	"	Good	
41		14	8 56	+6	10947	7955.376	+ 2	3.5	"	Fair	
42		Mar. 25	10 28	+3	11373	8026.438	− 1	0.9	"	Fair	
43		Apr. 15	10 56	+2	11499	8047.457	− 2	2.3	"	Fair	
44		26	11 18	0	11565	8058.471	+ 2	3.5	"	Good	
45		May 3	11 28	−1	11607	8065.477	+ 2	3.5	"	Good	
46		14	11 44	−2	11673	8076.487	+ 2	3.5	"	Good	
47	1910	Apr. 4	6 44	+2	15808	8766.282	− 2	2.3	H	Good	*Riv. di Astr.*, **4**, 310
48		6	6 43	+2	15820	8768.281	− 5	6.6	"	Good	
49		7	6 58	+2	15826	8769.292	+ 5	7.8	"	Fair	
50	1911	July 9	9 2	−6	18572	9227.372	− 2	2.3	L	Good	*A.N.*, 4542
51	1913	June 25	9 11	−5	22870	9944.379	+ 15	21.6	P	Uncertain	*Mem. Sp. It.*, **2**, 193
52		July 1	9 21	−6	22906	9950.385	+ 15	22.4	"	Fair	
53		3	8 53	−6	22918	9952.366	− 6	8.0	"	Fair	
54		4	9 2	−6	22924	9953.372	− 1	0.9	"	Good	
55		5	9 2	−6	22930	9954.372	− 1	0.9	"	Fair	
56		6	8 40	−6	22936	9955.357	− 17	22.5	"	Uncertain	
57		7	8 54	−6	22942	9956.366	− 9	12.4	"	Uncertain	
58		8	9 15	−6	22948	9957.381	+ 5	7.8	"	Uncertain	
59	1915	Jan. 28	17 19	+6	26361	0526.726	− 5	7.5	J	....	Letter
60		29	17 22	+6	26367	0527.728	− 4	5.9	"	....	
61	1916	Apr. 26	19 25	0	29083	0980.809	− 5	7.3	Sh	Good	This paper
62		29	11 28	0	29099	0983.478	− 6	8.0	Bi	Fair	
63		May 11	11 48	−2	29171	0995.490	− 5	6.6	"	Good	
64		17	11 53	−2	29207	1001.494	− 6	8.0	"	Good	
65		19	11 54	−2	29219	1003.494	− 8	11.0	"	Good	
66		21	11 59	−3	29231	1005.497	− 7	9.5	"	Good	
67		28	12 14	−3	29273	1012.508	−0.002	2.3	"	Fair	

* Good. † Uncertain; minimum phase very flat. ‡ Uncertain. § Uncertain; light-curve discordant.

used only to indicate the representation in 1915 by the adopted elements.[1]

For the designation of the observers the following abbreviations are used, the method of observation being indicated in parentheses: MK = Müller and Kempf (photometric); F = Fauth (estimates); PJ = Parkhurst and Jordan (photographic); T = Tikhoff (photographic); Ba = Baldwin (photometric); H = Horn (photographic); L = Lazzarino (photometric); P = Padova (photometric); J = Jordan (photographic); Sh = Shapley (photographic); Bi = Van der Bilt (photometric).

The following points are to be noted:

1. The minima published by Müller and Kempf and those by Parkhurst and Jordan have been taken as they stand. There are two misprints in the latter's comparison with Müller and Kempf's data;[2] the calculated time of the minimum of December 7 should be $19^{h}55^{m}$, and the residual minus instead of plus.

2. The detailed observations of Fauth, made by Argelander's method, are not accessible. Though the results are probably less accurate than those obtained by photometric methods, we have not thought ourselves justified in rejecting them on account of this a priori assumption.

3. All the observations made by Tikhoff have been plotted and several minima derived which had not been discussed previously. Though the various filters seem to give systematic differences in the time of minimum, the accompanying tabulation, based on data

Color Screen	Residuals (Unit is $0^{d}0001$)	Mean Residual
Orange	+23, −33, − 1, +15, −29	− 5
Green-blue	+6, +46, +38	+30
Ultra-violet	+34, +34, +46, +43, +33	+38

[1] For the sake of completeness, times of minima have also been computed for the new Harvard photographic observations from the data given by Russell, Fowler, and Borton in Table IX, *Astrophysical Journal*, **45**, 321, 1917. The representation of these times by our adopted light-elements is as follows:

Mean Epoch	1,120	7,380	16,000	24,600
O.−C.	$+0^{d}0015$	$+0^{d}0008$	$+0^{d}0016$	$-0^{d}0004$

[2] *Astrophysical Journal*, **23**, 84, 1906.

in Table IV, shows that these differences hardly exceed the observational uncertainty.[1] The individual measures have a considerable accidental error, and as only one of the color-filters was used at each minimum systematic errors may be involved if the period is not strictly uniform.

4. All the observations by Baldwin have also been plotted, and with the aid of the mean light-curve given by him several new minima were derived from incomplete light-curves and are available for the discussion.

5. Horn, working with photographic trails, gives the times of three maxima. Since it is uncertain whether the minima occur exactly half-way between the maxima, and since the latter can usually not be determined with the same accuracy as the minima, we have made a new grouping of Horn's observations for an independent derivation of the minima. A striking feature of his mean light-curve, entirely differing from the results of other observers, is that the horizontal line through the points of median magnitude is divided by the light-curve into approximately equal parts. The curves obtained by other observers all indicate that the part of this line below the maximum is nearly three times as long as that above the minimum. The actual figures are:

Müller and Kempf..........	$0^d.125$	and	$0^d.042$
Baldwin....................	0.121	"	0.046
Van der Bilt................	0.125	"	0.042
Horn.......................	0.097	"	0.070

Further, one (and probably two) of Horn's observed curves shows a constant minimum of more than 25 minutes. The explanation of both phenomena may be that the photographic trails at minimum were so faint that they could not be measured accurately.

6. The mean light-curve published by Padova was derived from a very non-homogeneous grouping. A new mean curve has been constructed and the separate minima determined as accurately as the observations allow.

[1] For a note on the interpretation of Tikhoff's results see Kron, *Potsdam Publication*, **22**, Pt. 3, p. 56, 1912.

The 64 minima used in the computation yield the following new light-elements:

Heliocentric Minimum = J.D. 2418075.8176 + $0^d.1668196E$, Gr.M.T.,

the period of revolution being double the interval between minima.

The residuals to which the foregoing formula leads are given, both in fractions of a day and in minutes of time, in the sixth and seventh columns. The mean residual is $0^d.0050$. The algebraic sum of the residuals is $-0^d.0014$. There are 30 positive and 34 negative residuals, 37 recurrences of sign, and only 26 changes, which points to some systematic change in the period. The evidence for this is strengthened by the fact that the first and the last six residuals are all negative, with practically the same mean value, viz.:

Mean epoch =	148,	Mean residual =	$-0^d.0066$
" " =	29,200,	" " =	-0.0053.

Were the time of normal minimum displaced by this amount, all but four of the intermediate residuals would become positive.

There is, however, no indication of a systematic difference between the odd and even minima. We derive:

Mean residual of 39 even minima = $+0^d.0005$
Mean residual of 25 odd minima = -0.0009

a difference wholly covered by the observational errors.

From the present study a perturbation of some character appears certain, but the data are as yet insufficient to determine the law of variation.

SUMMARY

1. Mount Wilson photographic observations of W Ursae Majoris, the star of highest known density, give a color-index in agreement with its spectral type (G), and show no conspicuous change of color between minimum and maximum light.

2. From a series of photometric measures made at the Utrecht Observatory (Table III) the times of six minima have been derived.

3. A study of 64 minima, distributed throughout an interval that includes 29,000 recurrences of minimum light, reveals a small but distinct perturbation of the period (Table IV).

Mount Wilson Solar Observatory
June 1917

BAXANDALL'S EXPLANATION OF AN "ABNORMAL" SOLAR SPECTRUM[1]

By GEORGE E. HALE

The following letter from Mr. Baxandall gives a simple and adequate explanation of two "abnormal" photographs of the solar spectrum in the neighborhood of a sun-spot, taken at the Kenwood Observatory in 1894, and described in this *Journal* in 1902.[2] The photographs in question were among several made in parallel strips on a single photographic plate, for the purpose of ascertaining the exposure times required for spectra of different orders. The slit of the spectrograph happened to lie across a sun-spot in the 2-inch solar image given by the Kenwood refractor, and most of the photographs on the plate show the usual bright reversals of H and K over the spot region. In two of them, however, these reversals were absent, and the spectra were so changed in appearance as to be unrecognizable. The idea that their peculiarities might be due to the chance superposition of two spectra of different orders was entertained at the time, but the absence of the bright H and K reversals where they should have appeared, together with other causes, unfortunately led us to dismiss this explanation. Finally, after much hesitation because of the improbable character of the spectrum, and after copies of the photographs had been sent to a number of spectroscopists in the hope that an explanation could be found, I published the photographs, together with measures of the lines and estimate of their intensities by Dr. Adams. These showed that most of the lines of the "abnormal" spectra were solar lines in Rowland's table, greatly changed in relative intensity. One of the "abnormal" spectra was intermediate in character between the solar spectrum and the other "abnormal" spectrum, and both showed two unknown bright lines, which, with other features mentioned by Mr. Baxandall, should

[1] *Contributions from the Mount Wilson Solar Observatory*, No. 143.

[2] "Solar Research at the Yerkes Observatory," *Astrophysical Journal*, **16**, 220, 1902.

have furnished us the clue so effectively recognized and applied by him. The fact that the apparent wave-lengths of these bright lines changed appreciably between the two exposures helped to mislead me, but I shall not attempt to offer any excuse for being so easily deceived.

Mr. Baxandall's letter is as follows:

SOLAR PHYSICS OBSERVATORY
Cambridge, June 4, 1916

DEAR PROFESSOR HALE:

It is only after considerable hesitation that I have decided to send you the present communication. It relates to your paper of 1902, containing an account of an apparently anomalous solar spectrum which has been considered to be due to a fleeting upheaval in the solar atmosphere. Doubtless, in common with many others engaged in spectroscopic investigations, I have always felt somewhat sceptical as to the reality of the phenomenon to which the abnormality of the spectrum was supposed to be due, although I have never been able to explain the strange spectra from any other point of view. My unbelief was based mainly on the following heads:

1. The isolated nature of the type of abnormality shown.

2. The restricted part of the spectrum in which it revealed itself.

3. The fact that there was no kind of lead given by the type of line or chemical elements involved.

I have on many occasions referred to your plates in the hope of getting some light on the real origin of the peculiar spectra, but with no success, until on Friday last, while examining the spectra again, I became conscious of the fact that the grouping of many of the most outstandingly strong lines in the abnormal spectrum had a familiar aspect, and the aspect was that of some of the chief landmarks in the solar spectrum itself. Thus the three strong lines 3921.87, 3927.77, and 3930.45 of the abnormal spectrum and the weaker bunch of lines 3916.84–3918.50 to the left, struck me as being uncommonly like the Fe triplet 4045, 4063, 4072, and the manganese bunch 4030–4036 of the normal solar spectrum. Then it dawned on me that probably by some chance circumstance, a small-dispersion solar spectrum had been superposed on your third-order spectra. Measuring on your reproduction the range of the three strong lines mentioned, I found it was exactly one-third the space occupied by the three Fe lines 4046–4072 on your third-order spectra. This suggests that the superposed spectrum was one of the first order of the grating used in your research. Employing this measured interval between your abnormal lines 3921.87, 3930.45 as a unit, and working backward on your spectra, I found that the two strange narrow bright lines 3884.64, 3896.21 agree in position with the H and K lines of the superposed spectrum. As these bright lines apparently extend equal distances on each side of the spot band, they are probably reversals of H and K in the added spectrum.

Working in the same way to the right, and taking the most outstanding line λ 3981 of your list, this revealed itself as the 4226.9 (Ca) line of the superposed spectrum. As the same proportion of one to three held in every case where intervals between solar landmarks were considered, it seems to me that the evidence is very conclusive that the strengthened lines in the abnormal spectrum are really due to the strong and outstanding lines of the small-scale superposed spectrum. The fact that it is only over a restricted region of your large-scale spectra where the abnormalities occur is also in favor of the contention that a small-scale superposed spectrum is involved.

As the intermediate spectrum shows a far closer resemblance to the normal solar spectrum, I have not thought it worth while to investigate in a similar manner to the above the irregularities in that spectrum, especially as, if it is satisfactorily established that any superposition of spectra has taken place, the whole argument for an upheaval in the solar photosphere must of necessity fall to the ground.

I feel sure that if a solar spectrum is prepared of such a scale that the lines 4071.9 Fe, 4226.9 Ca, on it are the same interval apart as the abnormal lines 3930.45, 3981.98 of your reproduction, and the strips so adjusted that these two pairs fit, the counterparts of the outstanding groups and lines of the normal spectrum will be apparent in your reproduced abnormal spectra along the whole range of the superposed spectrum. A few of the normal groups and lines will doubtless be modified in intensity where an outstandingly strong line of your "real" third-order spectrum blends with a line or group in the superposed spectrum.

I append a short list of lines which I consider, from rough measurements on the reproductions, to be identical: that is, the apparently abnormal lines of the reproduction, which I think are the real lines of the superimposed spectrum.

After I had come to the conclusion expressed above, I noticed that you mention in your paper having a suspicion at the time of the investigation that superposed spectra might be the cause of the apparent anomaly, but that after consideration you had come to the conclusion that that view was untenable. There is now little or no doubt in my mind that this is the real explanation. At the same time, I concede that, if you still have the original plates, you have the best opportunity of testing the points I raise.

I may mention just one other point. If the two bright narrow lines are not H and K reversals of the added spectrum, what are the chances against your measured wave-length interval between the two (when assumed to be real lines in the third-order spectra) being exactly one-third of the real wave-length interval between H and K? Thus your mean interval between the lines is 11.635 $\mu\mu$. One-third the interval between H and K is 11.600 $\mu\mu$. The difference between these two is probably within the errors of measurement.

Yours very truly,

F. E. BAXANDALL

All the lines in the first column are well-strengthened lines in the abnormal spectrum, and all those in the second column are conspicuous lines in the

Strengthened Lines in "Abnormal Spectrum"	Corresponding Lines in Superposed Spectrum	
3884.67 (bright)	3933.83 K (Ca) reversal	
3896.21 "	3968.63 H (Ca) "	
3908.41	4005.41 Fe	
3908.90	4007.	Narrow solar group
3916.84 to 3918.50	4030.9 to 4035.9	Mn group
3921.87	4045.98 Fe	Well-known triplet
3927.77	4063.76 Fe	
3930.45	4071.91 Fe	
3972.61	4198.40 to 4199.27	Narrow solar group
3973.74	4202.20 Fe	
3981.98	4226.90 Ca	
3989.91	4250.29 Fe 4250.95	
3993.05	4260.15 Fe 4260.64	
3996.80	4271.33 Fe 4271.93	
4005.41 to 4011.03 wide hazy group	4299. to 4315.	Fraunhofer's "g" group
4013.95	4324.	Narrow solar group
4014.70	4325.94 Fe	
4033.81	4383.72 Fe	Well-known triplet
4040.80	4404.93 Fe	
4044.19	4415.29 Fe	

normal solar spectrum. If any two of the former be taken it will be found that the interval between them is approximately one-third the interval between the corresponding two of the latter. The chance against this being a fortuitous result seems to me to be "beyond arithmetic."

As a further check we may compute the approximate wave-lengths of the solar lines corresponding to lines of the "abnormal" spectrum, on the assumption that a first-order spectrum is super-

posed on the third order, λ 3921.87 of the third order being coincident with λ 4045.98 of the first order.

Abnormal	Calculated First Order	O.−C.
3884.5 bright......	3933.3 (K)	+0.5
3896.1 "	3968.3 (H)	+0.3
3927.8............	4063.8	0.0
3930.4............	4071.9	0.0
3940.3............	4101.6	+0.3
3950.5............	4132.3	−0.1
3954.4............	4144.0	0.0
3982.0............	4227.2	−0.3
4014.7............	4325.9	0.0
4033.8............	4383.5	+0.2
4040.8............	4404.6	+0.3
4044.2............	4414.8	+0.5

Evidently no further evidence is needed to establish the truth of Mr. Baxandall's conclusions.[1]

Mount Wilson Solar Observatory
October 8, 1917

[1] Absence from the Observatory and the pressure of other duties have unduly delayed the re-examination of the original photographs and the publication of this note, which has experienced the fate of several other astronomical manuscripts, essentially completed many months ago, but still unpublished.

THE MINIMUM RADIATION VISUALLY PERCEPTIBLE

By HENRI BUISSON

Attention has been called to this subject in the two recent papers by H. E. Ives[1] and by H. N. Russell.[2] In closing his paper Russell expressed the desire that experiments on this point should be made in the laboratory.

I recently had occasion to make some determinations of the limits of sensibility of the eye, and I believe that it would be interesting to communicate these results in spite of the fact that they are not very numerous or extensive.

I undertook photometric measures on phosphorescent screens similar to those employed to make visible in darkness certain objects, such as the hands of a watch. These screens are covered with ZnS rendered luminous by the addition of a salt of radium. These are very faint, and, since their color corresponds nearly to the maximum of sensibility of the eye in the spectrum, they readily serve for determining the limit of sensibility.

The photometric measures were made as relative comparisons, but it has been easy to modify them in connection with absolute measures. The phosphorescent screen was compared with a film diffusing by transmission, which had been previously studied, so that its light was known when it was illuminated by a source of known intensity. It was thus found that the brightness of the phosphorescent screen varied, according to the screen, from 2 to 4×10^{-6} candles per sq. cm.

The screens were circular disks of 2.5 mm and of 5 mm diameter and were examined in darkness at increasing distances, the maximum distance at which they were seen being measured. The necessary precautions were taken to avoid errors of imagination: an assistant moved the screen, and the observer announced the motion that had been made, if he perceived it; or the two disks were placed on the same sheet of cardboard, and the observer stated the direction of the line joining them.

[1] *Astrophysical Journal*, **44**, 124, 1916. [2] *Ibid.*, **45**, 60, 1917.

The observations have been made in only a limited number, and by a single person having ordinary vision, after remaining in darkness for about 15^m. Table I gives the results, together with the computed distance at which a candle would be still visible.

TABLE I

Diameter of Disk	Brightness in Candles per sq. cm	Intensity in Candles	Maximum Distance of Visibility	Distance of Visibility of a Candle
2.5 mm..........	1.7×10^{-6}	8.4×10^{-8}	7.7 m	26.5 km
5.06.............	4.15×10^{-6}	83.0×10^{-8}	26.0	23.5
5.12.............	2.77×10^{-6}	57.2×10^{-8}	20.0	26.4

The results indicate directly the maximum distance at which a candle is visible. They are, furthermore, in excellent accord, yielding a mean of 27 km. That distance is much greater than has heretofore been accepted (about 11 km). It should be stated, moreover, that an actual candle would not be visible at that distance, as much because of the atmospheric absorption as because of the Purkinje phenomenon, which is so appreciable with the red color of the candle.

It is also easy to express the results in terms of stellar magnitude: assuming that a candle at one km is of magnitude 0.82, we deduce that the limit of visibility would be a star of magnitude 8.0, a value very close to that given by Russell, but obtained by an entirely different method.

It should be added that the luminous surfaces used were seen, at the limit of visibility, under very small angles—that is to say, under conditions which are similar to those under which we observe the stars. If we express in energy the quantity of light received by the eye at the limit of visibility, adopting the same values that Russell did, namely, 1.59 ergs per second per sq. cm for an illumination of one lux, and 0.57 sq. cm for the area of the pupil dilated widely in the darkness (diameter = 8.5 mm), we shall obtain 1.25×10^{-9} ergs per second, a value which is a little higher than that given by Russell.

MARSEILLE
August 1917

SOLAR HYDROGEN "BOMBS"[1]

By FERDINAND ELLERMAN

Visual and photographic observations of a solar phenomenon which had previously escaped our attention have been carried on at this observatory during the past two years.

On September 21, 1915, while the writer was observing the Hα line for reversals and distortions in an active spot-group, there suddenly appeared a very brilliant and very narrow band extending four or five angstroms on either side of the line, but not crossing it. In a couple of minutes it faded away and was not seen again. A month later, on October 21, more observations were recorded and a spectrogram was secured.

On the first occasion the appearance was so extraordinary that it seemed hardly real; after the second observation, however, the existence of such phenomena as part of the solar activity seemed established, and a search has been made for them whenever conditions of seeing and other work have permitted.

There are two conditions essential for observation—good seeing and a large solar image—as the area of the phenomenon, even with the 16-inch image of the sun at the 150-foot tower telescope, is so small that only with difficulty is the point of disturbance kept on the slit.

The appearance of the phenomenon indicates something in the nature of an explosion, in which hydrogen seems to be the only element playing a part. The duration is only a few minutes—from one to three on the average, and from five to ten minutes rarely. This sudden performance suggested the name of hydrogen "bomb," which we have adopted to designate it.

In the *Astrophysical Journal*, **30**, 78, 1909, Dr. Walter M. Mitchell gives an account of solar observations made at Haverford College Observatory, together with a drawing which illustrates the appearance of Hα in the spectrum of a "bomb," and from his

[1] *Contributions from the Mount Wilson Solar Observatory*, No. 141.

description there seems to be no doubt of its being such. This is the only recorded observation of the kind that has come to our notice, and it must have been an unusually large one to have been seen with the instrument used by Dr. Mitchell. Furthermore, he mentions D_3 as showing the band, while our observations have failed to detect it.

While the average width of the $H\alpha$ band is about 8 A, in an exceptional and probably an extreme case it was as wide as 30 A. In general the band appears symmetrical on each side of the line, but in a few measured cases the extension was greater on the violet side than on the red. In none have we found greater extension on the red side.

The regions where the "bombs" are likely to appear are around and among active spot-groups, especially groups which are developing and composed of many members. In such groups they appear at the outer edge of the penumbra, or at points between the spots, and also at small distances to one side. When a "bomb" appears and fades away, the point of disturbance should be kept under observation, as it is very probable that others will appear in the same place. At times they seem to follow one another like the balls of a Roman candle at intervals varying from ten to twenty minutes or more.

They frequently appear in the faculae so that their spectra are superposed on those of the faculae, thus giving the appearance of great extension to the bright "bomb" band. On rarer occasions they are superposed on bright reversals of $H\alpha$ over eruptive regions, but this is an uncommon occurrence, and the distinction is easily made between the two phenomena by the flickering of the "bomb" band compared to the $H\alpha$ reversal, due to the effect of seeing.

The observations have been confined mainly to $H\alpha$, although D_1, D_2, D_3; b_1, b_2, b_4; $H\beta$ and $H\gamma$ have been examined. None but the hydrogen lines have shown the band. $H\beta$ has been photographed and shows the band fairly well, but not as well as $H\alpha$ nor of as great width. $H\gamma$ also shows the band, but less clearly than $H\beta$.

The level at which the "bombs" occur must lie well below the reversing layer, as the hydrogen absorption line is not affected by them, nor do they seem to affect the other Fraunhofer lines.

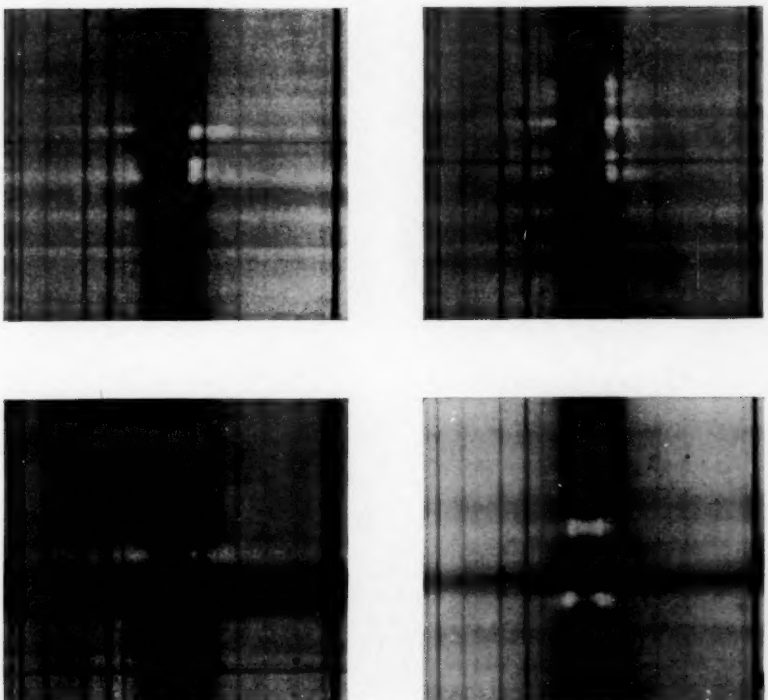
In Plate XVI, *a* and *b*, are shown spectra of "bombs" which appeared in regions where the $H\alpha$ line shows considerable distortion due to radial velocities of the hydrogen flocculi. It will be noted that the "bomb" bands do not extend as far on each side of $H\alpha$ as mentioned above, but this is to be expected, as it is impossible to guide and keep the image of the "bomb" on the slit perfectly during the entire exposure, and the fainter extensions which are easily observed visually are lost in the photograph. Plate XVI, *c*, shows the "bomb" band superposed on the spectrum of a facula, and in *d* are shown two bright reversals of $H\alpha$ near a small spot, to illustrate the difference in appearance between ordinary eruptive reversals of $H\alpha$ and the bright band due to "bombs."

Mount Wilson Solar Observatory
October 1917

PLATE XVI

a

b



c

d

a and *b*. Appearance of the Hα line in an active sun-spot region, showing the bands of "bombs" and the distortions of the line.

c. Spectrum of a "bomb" (close to the dark Hα line) superposed on the spectrum of a facula.

d. Appearance of reversals in an active region.

Reviews

The Principles of Aërography. By Alexander McAdie, Professor of Meteorology, Harvard University, and Director of the Blue Hill Observatory. Chicago and New York: Rand McNally & Co., 1917. Figs. 114. 8vo, pp. xvi+318. $3.00.

The usual book review generally falls into one of three fairly distinct classes. In the first place, there is the colorless review in which merely the number of pages, the number of illustrations, the headings of the various chapters, and similar information are given. Then there is the extremely eulogistic review which reads like an advertisement of the book and leads one to believe that it is the best which has been or could be written on the subject. Lastly, there is the extremely critical review in which faults of arrangement and sins of omission and commission are much emphasized. Should these be lacking, then typographical errors are called in, and, should these be in small number, then the incorrect spelling of a single foreign name is dwelt upon to such an extent that the book is made to appear inaccurate and the author unscholarly. The paragraphs which follow may suggest these three kinds of reviews.

The book under consideration consists of 16 pages of preliminary material, 278 pages of text, an appendix of 14 pages, and an unusually long index of 26 pages. It is divided into 18 chapters with the headings: "A Brief History of Meteorology"; "Units and Symbols"; "Temperature-Scales"; "Thermodynamics of the Atmosphere"; "Stratosphere and Troposphere"; "The Circulation of the Atmosphere"; "The Major Circulations"; "The Minor Circulations"; "Forecasting Storms"; "The Winds"; "The Water Vapor of the Atmosphere"; "Condensation"; "Dust and Microbes"; "Atmospheric Electricity"; "Precipitation"; "Floods and Notable Storms"; "Frosts"; "Solar Influences." There are 114 figures, of which a little less than half are illustrations. The rest are charts and diagrams, and they are all in the text. The appendix consists mostly of conversion tables for changing from one set of units to another. The index is simply an index and contains no biographical or additional information.

The title of the book, *The Principles of Aërography,* includes a word which will cause many to wonder if it contains anything different from

what would be expected if the title had been "The Principles of Meteorology." Frankly, it does not. Fifty or a hundred years ago the word "aërography" was perhaps as commonly used as the word "meteorology," but at present it is "meteorology" which is used for the treatment of all atmospheric phenomena, and it is too firmly intrenched to be easily forced to give ground. The author, however, would seem to make a distinction, for in the preface it is stated: "Thus aërography resembles geography in the larger sense, while meteorology, according to the general acceptation of the term, remains the science of recording diverse atmospheric conditions. The chief purpose of aërography is exploration with a view to utilizing the knowledge gained to insure human safety and to expedite progress."

The book is well printed on paper calculated to make the illustrations appear at their best. In fact, the illustrations are one of the chief charms of the book, for they are refreshingly new. They are not the common ones which have been so frequently used. The index is particularly well arranged and useful.

In reading the book one will be struck by the many things which deserve particular commendation. The treatment of the clouds is especially good and well illustrated. Two chapters, comprising 46 pages, are practically given up to them, and, in accord with the author's statement in the preface, they have been treated from the point of view of origin rather than appearance. The long chapter on atmospheric electricity, which is devoted almost entirely to the consideration of the thunder shower and lightning, is again an excellent one. The pictures of lightning flashes taken with a stationary and with a revolving camera are extremely good, and the spectrum of lightning has been given a very extended discussion. It should perhaps be noted that there are many pages in this chapter quoted from an article by another writer which appeared about three years ago. Full credit is, however, always given. The first chapter, on a brief history of meteorology, is very interesting and well written. In the fifth chapter, on the stratosphere and troposphere, will be found a particularly valuable summary of recent upper-air research, a subject which is very inadequately treated in any of the much-used textbooks on meteorology. The treatment of ice storms in chapter xv is far more complete and better illustrated than in any other book.

To what class of readers will the book particularly appeal? There are, in general, three classes of readers to whom a book on meteorology may appeal. These are, first, the general reader of scientific tastes;

second, the college undergraduate or advanced high-school student who is using the book as a textbook; third, the advanced student or reader who has already carefully studied other books on the subject. The book under consideration has numbered paragraphs, each with a definite heading, and marginal comments on the sides of the pages. All these are earmarks of the textbook and lead one to think that the author hoped it might be so used. Judged as a textbook, and thus the first book to be used by a student or general reader, it has many shortcomings. The material is not well arranged; there are too many omissions and not enough elementary detail in the treatment of many subjects. A single example will suffice. There are practically no descriptions or illustrations of the apparatus used for making meteorological observations. But the author says in the preface: "However, no attempt is made to reproduce *in extenso* weather charts and photographs of common instruments; the former are given in official reports and the latter belong more appropriately to the catalogues issued by instrument makers." But to how many readers of the book will a file of the catalogues of instrument-makers be accessible? The author states in the preface: "It is thought advisable that an effort be made to present this new knowledge [of about the last ten years] in a convenient form, even if considerably condensed. . . . . The present book, therefore, aims to give prominence to recent work that has been done in exploration of the air. . . . . Another important reason for offering this volume is the desire to further the use of the c.g.s. system of units. Throughout the book preference is given to absolute units, in the hope that the student will forget as soon as possible the old, arbitrary, and irrational units." The book, then, is to be considered as a reference work for recent research and as an advocate of absolute units. Viewed in this light, the book is all too short and many subjects deserve a more extended treatment. Weather prediction is covered in a single chapter of ten pages, and two of these are given up to a description of tornadoes and two to waterspouts—elementary and not recent material. Practically nothing is said about the weekly forecast of the Weather Bureau or long-range prediction. The semi-permanent highs and lows, often called centers of action, receive fairly brief treatment, as does the whole subject of correlation. Atmospheric optics, including halos and related phenomena, are treated in two pages and that in the chapter on dust and microbes—a grouping of subjects destined to fill one at first sight with dismay. It should, however, be said that in any book an author always expands the things in which he is especially interested, and a reviewer looks for those things in which he

is interested and judges the book by their presence or absence. Recent books and articles are frequently mentioned in the text, but one misses a classified bibliography at the end of each chapter or at the end of the book. The insistent use of absolute units will meet the warm approbation of a few. It will be a matter of indifference or annoyance to the majority of the readers of the book. There is as yet no perfect international agreement as to what absolute units are to be used or what they shall be called. The book thus lays stress on what is still more or less of a controversial point.

In conclusion, it can be stated without hesitation that the book as a whole is a very valuable contribution to meteorological literature, and this would be expected from the position and wide experience of the author. If a list of the ten best modern books on meteorology in English were being made, it would without question be included. How high it would stand in such a list would be a matter of individual judgment.

W. I. M.